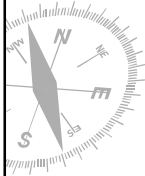


Math 119 – Plane Geometry

Sections 1.5 and 1.6
Intro to Proof and
Perpendicular Lines



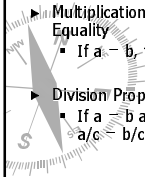
Logic Exercise

- ▶ Suppose that during a trial it is established that whoever committed the crime is color-blind and that Mr. Green and Miss Pink are two of the main suspects.
 - If the prosecutor produces proof that Mr. Green is color-blind, must the jury conclude that he is guilty?
 - If Miss Pink's lawyer proves that she is not color-blind, does it follow that she must be innocent?



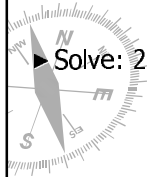
Review of Algebraic Properties

- ▶ Addition Property of Equality
 - If $a = b$, then $a + c = b + c$
- ▶ Subtraction Property of Equality
 - If $a = b$, then $a - c = b - c$
- ▶ Multiplication Property of Equality
 - If $a = b$, then $ac = bc$
- ▶ Division Property of Equality
 - If $a = b$ and $c \neq 0$, then $a/c = b/c$
- ▶ Distributive Property
 - $a(b + c) = ab + ac$
- ▶ Substitution Property
 - If $a = b$, then a replaces b in any equation.
- ▶ Transitive Property
 - If $a = b$ and $b = c$, then $a = c$.



Using the Properties

- ▶ Which property of equality justifies each conclusion?
 - If $2x - 3 = 7$, then $2x = 10$.
 - If $2x = 10$, then $x = 5$.
- ▶ Solve: $2(x - 3) + 4 = 10$ for x .



Our First Proof

► *Given:* $2(x - 3) + 4 = 10$

► *Prove:* $x = 6$

PROOF	
Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

How do we know when to stop?

Drawing Conclusions

► Given that $\angle 1$ and $\angle 2$ are complementary, then $m\angle 1 + m\angle 2 = 90$

- What property/definition justifies the conclusion?

► Given: A-M-Y

- What conclusion follows using the Segment-Addition Postulate?

► Given: $\angle 1$ and $\angle 2$ are supplementary

- What conclusion follows using the definition of supplementary angles?

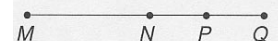
Try 1.5 #7-20

Given: A-P-B on \overline{AB}
Prove: $AP = AB - PB$



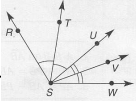
PROOF	
Statements	Reasons
1. A-P-B on \overline{AB}	1. Given
2.	2.
3. $AP = AB - PB$	3.

Given: $MN > PQ$
Prove: $MP > NQ$



PROOF	
Statements	Reasons
1.	1. Given
2. $MN + NP > NP + PQ$	2. Addition Property of Inequality
3.	3. Segment-Addition Postulate
4.	4.

Given: \overrightarrow{ST} bisects $\angle RSU$
 \overrightarrow{SV} bisects $\angle USW$
 Prove: $m\angle RST + m\angle VSW = m\angle TSV$



PROOF	
Statements	Reasons
1. \overrightarrow{ST} bisects $\angle RSU$	1.
2. $m\angle RST = m\angle TSU$	2.
3.	3. Same as reason 1.
4.	4. Same as reason 2.
5. $m\angle RST + m\angle VSW = m\angle TSU + m\angle USV$	5.
6. $m\angle TSU + m\angle USV = m\angle TSV$	6.
7.	7.

Try 1.5 #17-26

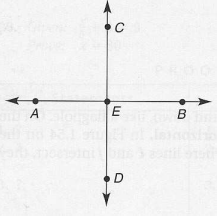
Perpendicular Lines

► **Def: Perpendicular lines** are two lines that meet to form congruent adjacent angles.

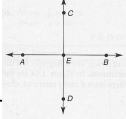
- Denoted: \perp

► What kind of angles do perpendicular lines form?

- Why is that true?



Given: $\overline{AB} \perp \overline{CD}$,
 intersecting at E
 Prove: $\angle AEC$ is a right angle



PROOF	
Statements	Reasons
1. _____	1. _____
2. $\angle AEC \cong \angle CEB$	2. Definition of _____
3. $m\angle AEC = m\angle CEB$	3. Definition of _____
4. $\angle AEB$ is a straight angle and $m\angle AEB = 180$	4. Measure of a straight angle equals _____
5. $m\angle AEC + m\angle CEB = m\angle AEB$	5. _____ Postulate
6. $m\angle AEC + m\angle CEB = 180$	6. _____
7. $m\angle AEC + m\angle AEC = 180$ or $2m\angle AEC = 180$	7. _____
8. $m\angle AEC = 90$	8. _____
9. $\angle AEC$ is a right angle	9. Definition of _____

Try 1.6 #1-4, 11, 12


Relations

► A **relation** "connects" two elements of an associated set of objects. (Denoted: R)

► Examples:

- "is equal to"
- "is greater than"
- "is perpendicular to"
- "is complementary to"
- "is congruent to"
- "is a brother of"

► What objects are related in each case?



Properties of Some Relations

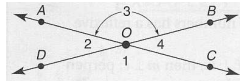
- ▶ **Reflexive Property:** $a R a$
 - Ex: $3 = 3$
 - Ex: $\angle A \cong \angle A$
- ▶ **Symmetric Property:** If $a R b$, then $b R a$.
 - Ex: If $x = 2$, then $2 = x$.
 - Ex: If $\angle 1$ is complementary to $\angle 2$, then $\angle 2$ is complementary to $\angle 1$.
- ▶ **Transitive Property:** If $a R b$ and $b R c$, then $a R c$.
 - Ex: If $x = y$ and $y = 4$, then $x = 4$.
 - Ex: If $AB \cong MN$ and $MN \cong XY$, then $AB \cong XY$.
- ▶ Is the relation "is less than" reflexive? symmetric? transitive?

Equivalence Relations

- ▶ A relation that is reflexive, symmetric, and transitive is an **equivalence relation**.
- ▶ Why is $=$ an equivalence relation?
- ▶ Verify that \cong is an equivalence relation by checking that it is reflexive, symmetric, and transitive.

Try 1.6 #13-18

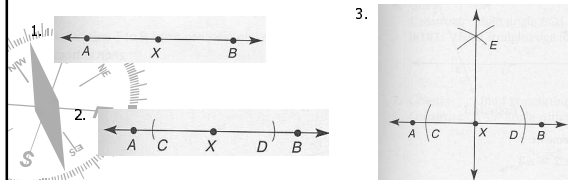
Given: AC intersects BD at O
Prove: $\angle 2 \cong \angle 4$



PROOF	
Statements	Reasons
1. _____	1. _____
2. $\angle AOC$ and $\angle DOB$ are straight angles with $m\angle AOC = 180$ and $\angle DOB = 180$	2. _____
3. _____	3. Substitution
4. $m\angle 1 + m\angle 4 = m\angle DOB$ and $m\angle 1 + m\angle 2 = m\angle AOC$	4. _____
5. $m\angle 1 + m\angle 4 = m\angle 1 + m\angle 2$	5. _____
6. _____	6. Subtraction Property of -
7. _____	7. Definition of Congruence
8. _____	8. _____

Constructing Perpendicular Lines

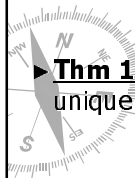
- ▶ How can we construct a line perpendicular to a segment that passes through the midpoint?
- ▶ Construct line perpendicular to a given line at a specified point on the given line (not necessarily the midpoint);



Try 1.6 #5-9

Perpendicular Line Theorems

► **Thm 1.6.1:** There is exactly one line perpendicular to a given line at any point on the line.



► **Thm 1.6.2:** The perpendicular bisector is unique.

Homework

► Due Thursday 6/17

- Study for Exam 1
- Suggested Preparation:
 - Read Sections 1.5 and 1.6

► 1.5: # 1-15 odd, 17-26

► 1.6: # 1-19

► Chapter 1 Review: # 1-14, 18-26, 30-35, 44, 46, 47

► Review Vocabulary/Theorems/Postulates

