

Math 119 – Plane Geometry

Sections 2.1 and 2.2
Parallel Lines 1 and Indirect Proof
6/21/2004

Example 1

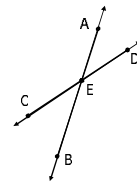
- Use definitions and theorems to complete each of the following:
1. If $\angle A$ and $\angle B$ are supplementary, then ____.
 2. If $\angle A$ and $\angle B$ are supplements of $\angle C$, then ____.
 3. If $\angle D$ and $\angle E$ are complementary, then ____.
 4. If $\angle D$ and $\angle E$ are complements of equal angles, then ____.
 5. If $\angle G$ and $\angle H$ are adjacent angles whose exterior sides form a line, then ____.
 6. If $\angle C$ and $\angle F$ are vertical angles, then ____.

Example 2

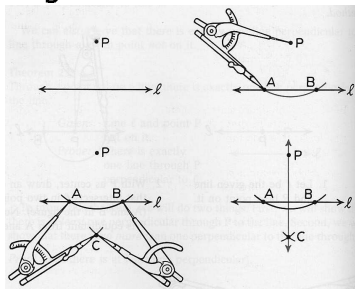
- Consider the following statement:
- "If $m\angle A = 30$, $m\angle M = 90$, and $m\angle Y = 60$, then they are supplementary."
1. Why would someone think this?
 2. Why is it wrong?
 3. Would it be correct to say $\angle M$ is complementary?

Example 3

- A-E-B, C-E-D, and $m\angle AED = x$
1. What can you conclude about EA and EB and about ED and EC?
 2. What conclusion can you draw about $\angle AED$ and $\angle CEB$?
 3. Find $m\angle CEB$.
 4. What conclusion can you draw about $\angle AED$ and $\angle DEB$?
 5. Find $m\angle DEB$.



Construction: Perpendicular Line Through Point Not on the Line

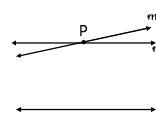


- **Thm 2.1.1:** From a point not on a given line, there is exactly one line perpendicular to the given line. (Proved later.)

The Parallel Postulate

- **Parallel Postulate:** Through a point not on a line, exactly one line is parallel to the given line.

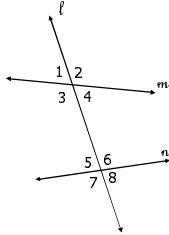
- This *cannot be proved* using theorems and postulates we have.



- **Q:** Given $n \parallel \ell$ and line m intersects line n at point P. Does it follow that line m must intersect line ℓ ?

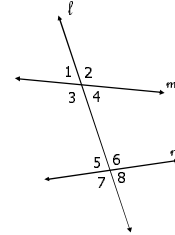
Parallel Lines - Definitions

- ▶ **Parallel lines** are lines in the same plane that do not intersect.
- ▶ A **transversal** is a line that intersects two (or more) other lines at distinct points.
- ▶ Angles that are formed between the two lines are **interior angles**.
- ▶ Angles that are formed outside the two lines are **exterior angles**.
- ▶ **Q:** Do the lines need to be parallel to have transversals/exterior angles/interior angles?



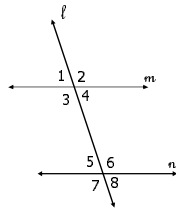
Transversals & Special Pairs of Angles

- ▶ **Corresponding angles** – A pair of angles that lie in the same relative position when two lines are cut by a transversal.
- ▶ **Alternate Interior Angles** – A pair of interior angles that have different vertices and lie on opposite sides of the transversal.
- ▶ **Alternate Exterior Angles** – A pair of exterior angles that have different vertices and lie on opposite sides of the transversal.
- ▶ **Q:** To how many angles does the phrase “corresponding angles” refer?



Parallel Lines and Congruent Angles

- ▶ **Post:** If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
- ▶ **Ex 4:** Suppose $n \parallel m$ and $m\angle 1 = 75$. Find the measures of the other angles.
- ▶ **Q:** Does intuition offer any conclusions about how the angles interact?

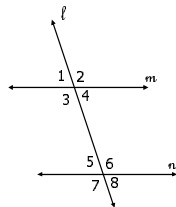


Theorems For Angles/Parallel Lines

- ▶ **Thm 2.1.2:** If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
- ▶ **Thm 2.1.3:** If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
- ▶ **Thm 2.1.4:** If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.
- ▶ **Thm 2.1.5:** If two parallel lines are cut by a transversal, then the exterior angles on the same side of the transversal are supplementary.
 - Let's prove 2.1.2 and 2.1.4 in class.

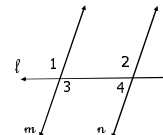
Example 5: Using the Theorems

- ▶ Given $n \parallel m$ with transversal l
 - There are 2 classes of congruent angles. Name the members of each class.
 - $m\angle 1 = (x + 4)(x - 3)$.
 $m\angle 8 = x^2 - 3$. Find x .
 - $m\angle 3 = 4x + y$. $m\angle 5 = 6x + 5y$.
 $m\angle 6 = 5x - 2y$. Find x and y .



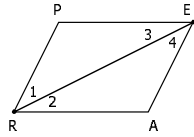
Example 6

- ▶ $n \parallel m$ with transversal l
 1. What are $\angle 1$ and $\angle 2$ called with respect to the lines?
 2. Why is $m\angle 1 = \angle 2$?
 3. What are $\angle 2$ and $\angle 3$ called?
 4. Why is $\angle 2 = \angle 3$?
 5. What are $\angle 3$ and $\angle 4$ called?
 6. Why are $\angle 3$ and $\angle 4$ supplementary?



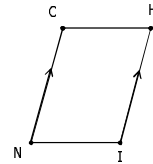
Example 7

1. Which angles are equal if $\overline{PE} \parallel \overline{RA}$?
2. Which angles are equal if $\overline{PR} \parallel \overline{EA}$?
3. Which angle is supplementary to $\angle PRA$ if $\overline{PE} \parallel \overline{RA}$?
4. Which angle is supplementary to $\angle P$ if $\overline{PR} \parallel \overline{EA}$?



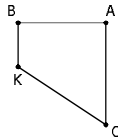
Example 8

- ▶ $\overline{CN} \parallel \overline{HI}$.
 - ▶ $m\angle C = (6x + 1)$
 - ▶ $m\angle H = (9x - 1)$
 - ▶ $m\angle I = 5(x + 3)$
1. Find x .
 2. Find $m\angle C$.
 3. Find $m\angle H$.
 4. Find $m\angle I$.



Example 9: Proof

- ▶ **Given:** $\overline{BA} \perp \overline{AC}$ and $\overline{BK} \parallel \overline{AC}$
- ▶ **Prove:** $\angle B$ is a right angle



Statements Related to the Conditional

- ▶ **Conditional:** If P, then Q.
 - Hypothesis – P
 - Conclusion – Q
- ▶ **Converse:** If Q, then P.
- ▶ **Inverse:** If not P, then not Q.
- ▶ **Contrapositive:** If not Q, then not P.

(Not P – $\sim P$ – negation of P)

Example 10

- Find P and Q.
 - State the converse.
 - State the inverse.
 - State the contrapositive.
- ▶ "If we study, we will make good grades."
 - ▶ "If Amy lives in Oakland, then Amy lives in California."
 - ▶ "If two angles are vertical angles, then they are congruent angles."

Find the truth values of each

Law of Negative Inference

1. $P \Rightarrow Q$
 2. $\sim P$
- } Premises
- C. $\therefore \sim Q$ } Conclusion

- ▶ Statements are either TRUE or NOT TRUE
- ▶ Example: Find the conclusion for the following argument:
 1. If a person lives in Los Angeles, then he/she lives in USA.
 2. Mary does not live in USA.

Example 11

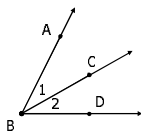
- ▶ Examine the validity of the following argument:
 - (1) If two angles are vertical angles, then they are congruent angles.
 - (2) Two angles are not vertical angles.
 -
 - (C) \therefore They are not congruent.

Indirect Proof

- ▶ To prove the statement "If P, then Q" by the indirect method:
 1. Suppose that $\sim Q$ is true.
 2. Reason from the supposition in step (1) until you reach a contradiction of a known fact.
 - Each statement beyond the supposition must be justified.
 3. Note that the supposition $\sim Q$ must be false and that Q is therefore true.

Example 12: Indirect Proof

- ▶ Given: \overline{BA} is *not* perpendicular to \overline{BD}
- ▶ Prove: $\angle 1$ and $\angle 2$ are *not* complementary

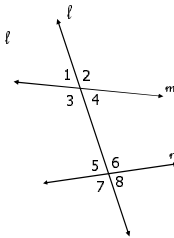


1. Suppose that $\sim Q$ is true
 - Usually the negation of your Prove
2. Reason until you reach a contradiction
 - Usually contradicting your Given
3. $\therefore \sim Q$ must be false and $\therefore Q$ is therefore true
 - Works since statements are TRUE or NOT TRUE

Example 13: Indirect Proof

- ▶ If two lines are cut by a transversal so that corresponding angles are not congruent, then the two lines are not parallel.

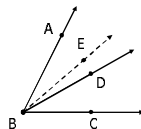
- ▶ Given: n and m with transversal l
 $\angle 1 \neq \angle 5$
- ▶ Prove: $n \not\parallel m$



Example 14: Indirect Proof

- ▶ The angle bisector of an angle is unique.

- ▶ Given: \overline{BD} bisects $\angle ABC$
- ▶ Prove: \overline{BD} is the only angle bisector for $\angle ABC$



Homework

- ▶ Due Tuesday 6/22
 - Read Sections 2.1 and 2.2
 - 2.1: #1-7, 9-17, 19-34
 - 2.2: #1-22