

# Math 119 – Plane Geometry

Sections 2.3 and 2.4  
Parallel Lines 2  
6/22/2004

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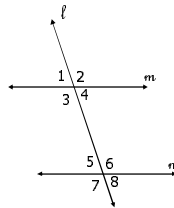
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## Example: Warm-up

- ▶ Name all angles congruent to  $\angle 1$ .
- ▶ Name all angles congruent to  $\angle 6$ .
- ▶ Write an equation relating  $\angle 3$  and  $\angle 5$ .



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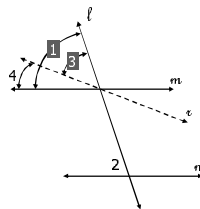
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**Theorem 2.3.1:** If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.

- ▶ Given:  $n$  and  $m$  with transversal  $l$   
 $\angle 1 \cong \angle 2$
- ▶ Prove:  $n \parallel m$



Use INDIRECT Proof

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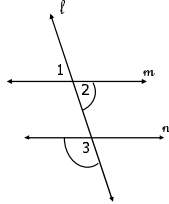
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**Theorem 2.3.2:** If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel.

► Given:  $n$  and  $m$  with transversal  $l$   
 $\angle 2 \cong \angle 3$



► Prove:  $n \parallel m$

► **Theorem 2.3.3:** If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel. (HW)

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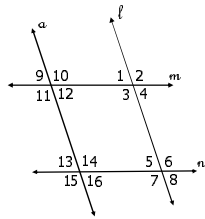
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### Example

► Which lines must be parallel if  $\angle 4 \cong \angle 5$ ?

► If  $\angle 9 \cong \angle 16$ , must  $\angle 1$  and  $\angle 10$  be supplementary?

► If  $\angle 6 \cong \angle 15$ , must  $\angle 1$  and  $\angle 10$  be supplementary?




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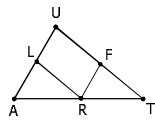
### Example

► Which line segments are parallel if...

1. ...  $m\angle LRF = m\angle RFT$ ?

2. ...  $\angle U$  and  $\angle UFR$  are supplementary?

3. ...  $m\angle A = m\angle FRT$ ?




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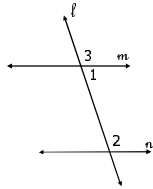
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**Theorem 2.3.4:** If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these lines are parallel.

- Given:  $n$  and  $m$  with transversal  $\ell$   
 $\angle 1$  is supplementary to  $\angle 2$



- Prove:  $n \parallel m$
- **Theorem 2.3.5:** If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel. (HW)

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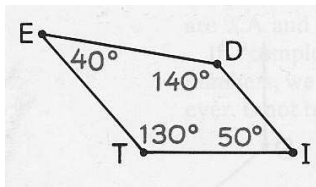
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### Example

- Which lines must be parallel?




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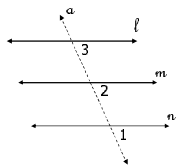
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**Theorem 2.3.6:** If two lines are each parallel to a third line, then these lines are parallel to each other.

- Given:  $\ell \parallel m$  and  $n \parallel m$  with transversal  $a$
- Prove:  $\ell \parallel n$




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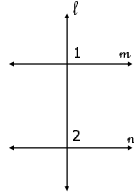
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**Theorem 2.3.7:** If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

- Given:  $m \perp \ell$  and  $n \perp \ell$
- Prove:  $m \parallel n$




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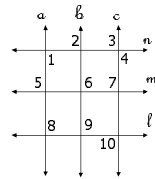
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### Example

- If possible, name a pair of lines that must be parallel if each of the following is true:

1.  $m\angle 1 = m\angle 3$
2.  $m\angle 2 = m\angle 5$
3.  $m\angle 3 = m\angle 7$
4.  $b \perp \ell$  and  $\ell \perp c$
5.  $m\angle 5 = m\angle 6$
6.  $\angle 1$  and  $\angle 8$  are supplementary
7.  $m\angle 3 = m\angle 4$
8.  $\angle 5$  and  $\angle 7$  are supplementary
9.  $m\angle 8 = m\angle 10$
10.  $a \perp n$  and  $b \perp m$




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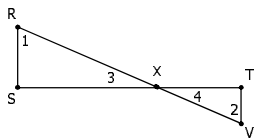
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### Example: A Proof

- Given:  $RV$  and  $ST$  intersect at point  $X$   
 $\angle 1$  and  $\angle 3$  are complementary  
 $\angle 2$  and  $\angle 4$  are complementary
- Prove:  $RS \parallel TV$




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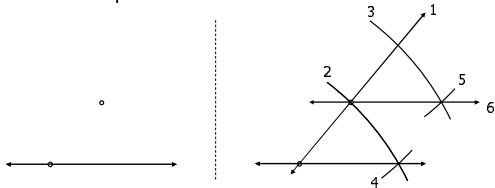
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## Construction: Parallel Lines

► Use the following theorem:

- If two lines are cut by a transversal so that corresponding angles are congruent, then these lines are parallel.




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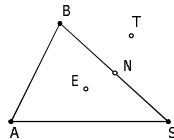
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## Triangles

► **Def:** A **triangle** is the union of three line segments that are determined by three noncollinear points.

- The points are the **vertices** of the triangle.
- The line segments are the **sides** of the triangle.

1. Name a point *on* the triangle, *in the interior* of the triangle, and *in the exterior* of the triangle.




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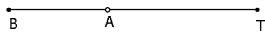
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## Example

► How do we know the union of line segments BA, BT and AT does not form a triangle?




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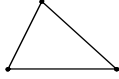
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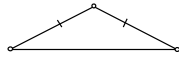
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## Triangles Classified By Sides

- ▶ A **scalene triangle** has no congruent sides.



- ▶ An **isosceles triangle** has two congruent sides.



- ▶ An **equilateral triangle** has three congruent sides.




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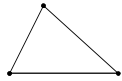
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## Triangles Classified By Angles

- ▶ An **acute triangle** has three acute angles.



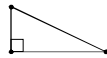
- ▶ An **obtuse triangle** has one obtuse angle.



- ▶ An **equiangular triangle** has three congruent angles.



- ▶ A **right triangle** contains one right angle.




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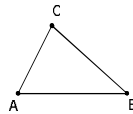
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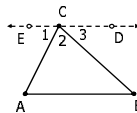
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**Theorem 2.4.1:** In a triangle, the sum of the measures of the interior angles is 180.

- ▶ Given:  $\triangle ABC$



- ▶ Prove:  $m\angle A + m\angle B = m\angle C$




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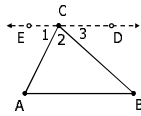
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## Auxiliary/Helping Lines

- ▶ When constructed, a helping line enables one to solve a problem or complete a proof.
- ▶  $\overleftrightarrow{ED}$  from the last proof was a helping line

### Justify via theorems:

- "Through a point outside a line, there is exactly one line parallel to the given line."
- Shorthand – "By construction."



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## Corollaries To Sum of Angles

- ▶ **Cor 2.4.2:** Each angle of an equiangular triangle measures  $60^\circ$ . (HW)
- ▶ **Cor 2.4.3:** The acute angles of a right triangle are complementary. (HW)
- ▶ **Cor 2.4.4:** If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
  - Discuss why true.

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## Example

- ▶ In  $\triangle TEA$ ,  $m\angle T = 10^\circ$  and  $m\angle E = 20^\circ$ .
  - Find  $m\angle A$ .
  - What kind of triangle is  $\triangle TEA$  with respect to its angles?
- ▶ In  $\triangle WEB$ ,  $m\angle W = 40^\circ$  and  $m\angle E = 70^\circ$ .
  - Find  $m\angle B$ .
  - What kind of triangle is  $\triangle WEB$  with respect to its angles?

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### Example

- ▶ In  $\triangle SAD$ ,  $\angle S \cong \angle A \cong \angle D$ .
  - What kind of triangle is  $\triangle SAD$  with respect to its angles?
  - What is the measure of each angle of  $\triangle SAD$ ?
- ▶ In  $\triangle WED$  and  $\triangle BOX$ ,  $\angle W \cong \angle B$  and  $\angle E \cong \angle O$ .
  - What can you conclude about  $\angle D$  and  $\angle X$ ?
- ▶ In  $\triangle LIE$ ,  $\angle L$  is a right angle.
  - What relation do  $\angle I$  and  $\angle E$  have?

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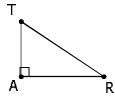
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### Example

- ▶  $\overline{TA} \perp \overline{AR}$ .
- ▶  $m\angle R = (3x + 5)$  and  $m\angle T = (4x - 6)$ 
  - Find  $x$ .
  - Find  $m\angle R$  and  $m\angle T$ .



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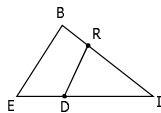
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### Example: Proof

- ▶ Given: In  $\triangle BIE$  and  $\triangle BRI$ ,  $m\angle B = m\angle DRI$
- ▶ Prove:  $m\angle E = m\angle IRD$



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### Example: Proof

► Given:  $\overline{NU} \perp \overline{UT}$

► Prove:  $\angle N + \angle T = 90$




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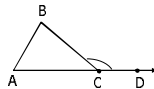
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### Exterior Angles

► When the sides of a triangle are extended, each angle that is formed by a side and an extension of the adjacent side is an **exterior angle** of the triangle.

► How does  $\angle DCB$  relate to  $\angle A$  and  $\angle B$ ? Why?




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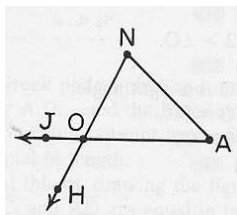
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### Exterior Angles – Example

► In this figure, JON and HOA are exterior angles

- Name a relation between angles JON and NOA
- Name a relation between angles JON and HOA
- Is JOH an exterior angle of the triangle?
- How many exterior angles does a triangle have at one vertex?
- How many exterior angles does a triangle have in all?




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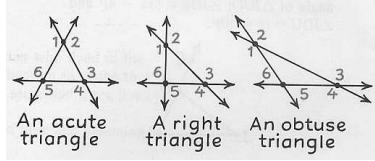
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### Example

► Use the following pictures to decide T/F:



- All six exterior angles of a triangle may be obtuse.
- Some of the exterior angles of an obtuse triangle are acute.
- All six exterior angles of a triangle may have different measures.
- An exterior angle of a triangle may be smaller than one of the nonadjacent interior angles of the triangle.
- An exterior angle of a triangle may be equal to one of the interior angles of the triangle.

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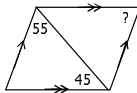
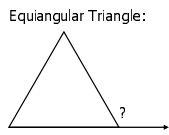
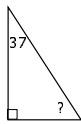
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### Example: Find the Angle




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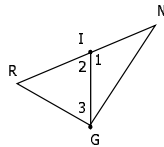
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### Example: Proof

- Given: R-I-N  
 $\angle 1$  and  $\angle 2$  are adjacent
- Prove:  $m\angle R = m\angle 1 - m\angle 3$




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## Homework

► Due Wednesday 6/23/2004

- Read Sections 2.3 and 2.4
- 2.3: #1-29, 31-33
- 2.4: #1-29, 34-40

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