Math 119 – Plane Geometry

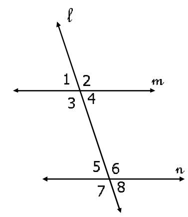
Sections 2.3, 2.4 and 2.5

Parallel Lines 2

6/22/2004

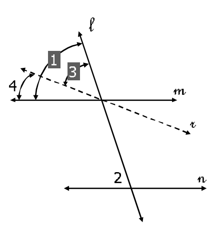
Example: Warm-up

- Name all angles congruent to $\angle 1$.
- Name all angles congruent to $\angle 6$.
- Write an equation relating $\angle 3$ and $\angle 5$.



Theorem 2.3.1: If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.

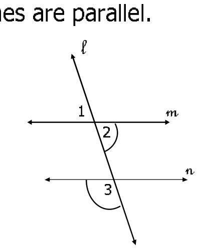
- Given: n and m with transversal ℓ; $\angle 1 \cong \angle 2$
- Prove: $n \parallel m$



Use INDIRECT Proof

Theorem 2.3.2: If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel.

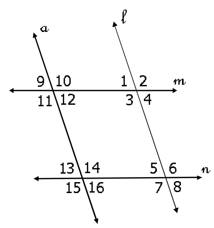
- Given: n and m with transversal ℓ; $\angle 2 \cong \angle 3$
- Prove: $n \parallel m$



- **Theorem 2.3.3**: If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel. (HW)

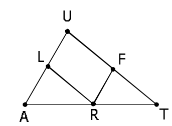
Example

- Which lines must be parallel if $\angle 4 \cong \angle 5$?
- If $\angle 9 \cong \angle 16$, must $\angle 1$ and $\angle 10$ be supplementary?
- If $\angle 6 \cong \angle 15$, must $\angle 1$ and $\angle 10$ be supplementary?



Example

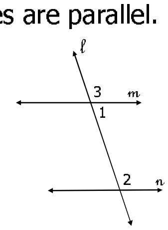
- Which line segments are parallel if…
  1. … m$\angle$LRF = m$\angle$RFT?
  2. … $\angle$U and $\angle$UFR are supplementary?
  3. … m$\angle$A = m$\angle$FRT?



Theorem 2.3.4: If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these lines are parallel.

- Given: n and m with transversal ℓ

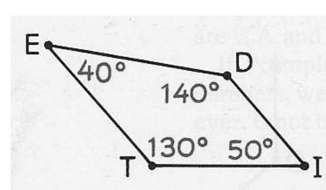
  $\angle 1$ is supplementary to $\angle 2$
- Prove: $n \parallel m$



- **Theorem 2.3.5**: If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel. (HW)

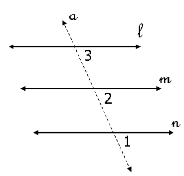
Example

- Which lines must be parallel?



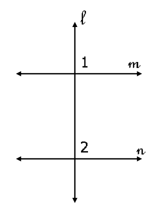
Theorem 2.3.6: If two lines are each parallel to a third line, then these lines are parallel to each other.

- Given: $\ell \parallel m$ and $n \parallel m$ with transversal a
- Prove: $\ell \parallel n$



Theorem 2.3.7: If two coplanar lines are each perpendicular to a third line, then these lines are parallel to each other.

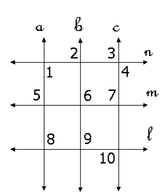
- Given: $m \perp \ell$ and $n \perp \ell$
- Prove: $m \parallel n$



Example

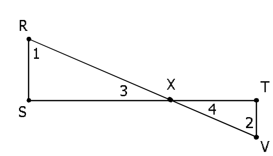
- If possible, name a pair of lines that must be parallel if each of the following is true:

1. m∠1 = m∠3
2. m∠2 = m∠5
3. m∠3 = m∠7
4. $b \perp \ell$ and $\ell \perp c$
5. m∠5 = m∠6
6. ∠1 and ∠8 are supplementary
7. m∠3 = m∠4
8. ∠5 and ∠7 are supplementary
9. m∠8 = m∠10
10. $a \perp n$ and $b \perp m$



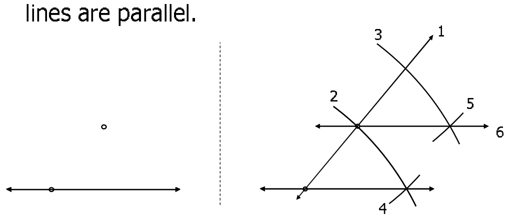
Example: A Proof

- Given: RV and ST intersect at point X
  ∠1 and ∠3 are complementary
  ∠2 and ∠4 are complementary
- Prove: RS ∥ TV



Construction: Parallel Lines

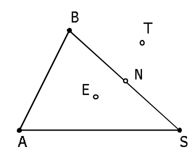
- Use the following theorem:
  - If two lines are cut by a transversal so that corresponding angles are congruent, then these lines are parallel.



Triangles

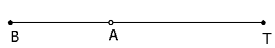
- **Def**: A **triangle** is the union of three line segments that are determined by three noncollinear points.
  - The points are the **vertices** of the triangle.
  - The line segments are the **sides** of the triangle.

1. Name a point *on* the triangle, *in the interior* of the triangle, and *in the exterior* of the triangle.



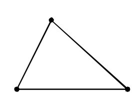
Example

- How do we know the union of line segments BA, BT and AT does not form a triangle?

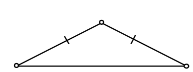


Triangles Classified By Sides

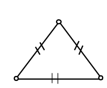
- A **scalene triangle** has no congruent sides.



- An **isosceles triangle** has two congruent sides.

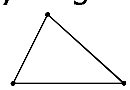


- An **equilateral triangle** has three congruent sides.

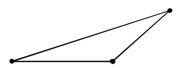


Triangles Classified By Angles

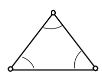
- An **acute triangle** has three acute angles.



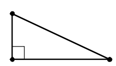
- An **obtuse triangle** has one obtuse angle.



- An **equiangular triangle** has three congruent angles.

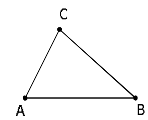


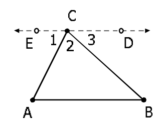
- A **right triangle** contains one right angle.



Theorem 2.4.1: In a triangle, the sum of the measures of the interior angles is 180.

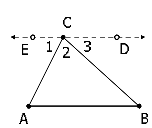
- Given: ΔABC
- Prove: $m\angle A + m\angle B = m\angle C$





Auxiliary/Helping Lines

- When constructed, a helping line enables one to solve a problem or complete a proof.
- $\overleftrightarrow{ED}$ from the last proof was a helping line
- Justify via theorems:
  - "Through a point outside a line, there is exactly one line parallel to the given line."
  - Shorthand – "By construction."



Corollaries To Sum of Angles

- **Cor 2.4.2**: Each angle of an equiangular triangle measures 60. (HW)
- **Cor 2.4.3**: The acute angles of a right triangle are complementary. (HW)
- **Cor 2.4.4**: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
  - Discuss why true.

Example

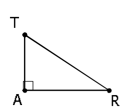
- In ΔTEA, m∠T = 10 and m∠E = 20.
  - Find m∠A.
  - What kind of triangle is ΔTEA with respect to its angles?
- In ΔWEB, m∠W = 40 and m∠E = 70.
  - Find m∠B.
  - What kind of triangle is ΔWEB with respect to its angles?

Example

- In ΔSAD, ∠S ≅ ∠A ≅ ∠D.
  - What kind of triangle is ΔSAD with respect to its angles?
  - What is the measure of each angle of ΔSAD?
- In ΔWED and ΔBOX, ∠W ≅ ∠B and ∠E ≅ ∠O.
  - What can you conclude about ∠D and ∠X?
- In ΔLIE, ∠L is a right angle.
  - What relation do ∠I and ∠E have?

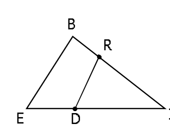
Example

- $\overline{TA} \perp \overline{AR}$.
- m∠R = (3x + 5) and m∠T = (4x – 6)
  - Find x.
  - Find m∠R and m∠T



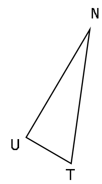
Example: Proof

- Given: In ΔBIE and ΔDRI, m∠B = m∠DRI
- Prove: m∠E = m∠IRD



Example: Proof

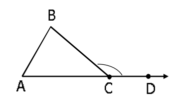
- ►Given: $\overline{NU} \perp \overline{UT}$
- ►Prove: ∠N + ∠T = 90



Exterior Angles

- ►When the sides of a triangle are extended, each angle that is formed by a side and an extension of the adjacent side is an **exterior angle** of the triangle.

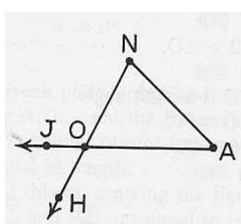
- ►How does ∠DCB relate to ∠A and ∠B? Why?



Exterior Angles – Example

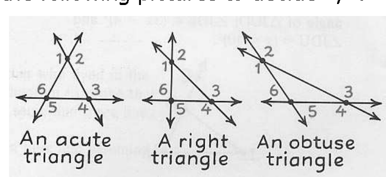
- ► In this figure, JON and HOA are exterior angles

a. Name a relation between angles JON and NOA
b. Name a relation between angles JON and HOA
c. Is JOH an exterior angle of the triangle?
d. How many exterior angles does a triangle have at one vertex?
e. How many exterior angles does a triangle have in all?



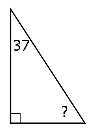
Example

- ► Use the following pictures to decide T/F:

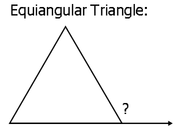


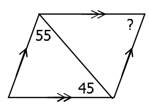
a. All six exterior angles of a triangle may be obtuse.
b. Some of the exterior angles of an obtuse triangle are acute.
c. All six exterior angles of a triangle may have different measures.
d. An exterior angle of a triangle may be smaller than one of the nonadjacent interior angles of the triangle.
e. An exterior angle of a triangle may be equal to one of the interior angles of the triangle.

Example: Find the Angle



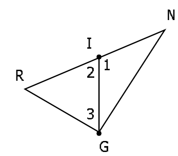
Equiangular Triangle:





Example: Proof

- Given: R-I-N
  ∠1 and ∠2 are adjacent
- Prove: m∠R = m∠1 – m∠3



Homework

- Due Wednesday 6/23/2004
  - Read Sections 2.3, 2.4 and 2.5
  - 2.3: #1-29, 31-33
  - 2.4: #1-29, 34-40