Math 119 – Plane Geometry

Sections 3.2 and 3.3

Triangles II

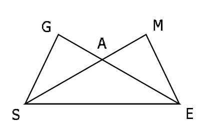
6/24/2004

Recall: Congruence of Triangles

- **Def**: Two triangles are **congruent** when the six parts of the first triangle are congruent to the six corresponding parts of the second triangle.
- Methods for Proving Congruence:
  - SSS
  - SAS
  - ASA
  - AAS
- **CPCTC**: Corresponding parts of congruent triangles are congruent.

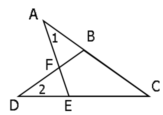
Example

- $\Delta SGE \cong \Delta EMS$
  a) SG = ____
  b) MS = ____
  c) SE = ____
  d) m∠G = ____
  e) m∠GSE = ____
  f) m∠ESM = ____



Suggestions: Proving Triangles ≅

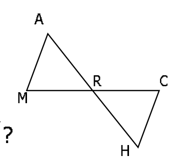
- Mark figures using the info from your givens
  - Right angles – *squares*
  - Congruent sides – *dashes*
  - Congruent angles – *arcs*
- Trace triangles to be proved congruent in different colors
- If triangles overlap, draw them separately



Example

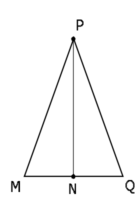
- ∠M and ∠C are supplements of ∠ARC
- MR = RC
- ∠ARM and ∠HRC are vertical angles

a) Why is ∠M ≅ ∠C?
b) Why is ∠ARM ≅ ∠HRC?
c) Why is ΔMAR ≅ ΔCHR?
d) Why is AR = RH?
e) Why is R the midpoint of $\overline{AH}$?



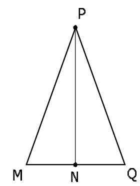
Example: Proof

- Given: $\overrightarrow{PN}$ bisects ∠MPQ
  $\overline{PM} \cong \overline{PQ}$
- Prove: $\overline{MN} \cong \overline{QN}$



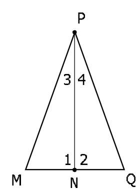
Example: Proof

- Given: $\overrightarrow{PN}$ bisects $\angle MPQ$
  $\overline{PM} \cong \overline{PQ}$
- Prove: $\overline{PN}$ bisects $\overline{MQ}$



Example: Proof

- Given: $\overline{PM} \cong \overline{PQ}$
  $\angle 3 \cong \angle 4$
  M-N-Q
- Prove: $\overline{PN} \perp \overline{MQ}$

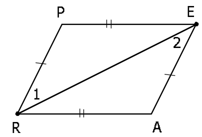


Triangle Proofs: 3 Types of Conclusions

- Proving triangles congruent
  - Use SSS, SAS, ASA, AAS
- Proving corresponding parts of congruent triangles congruent
  - Get triangles congruent using SSS, SAS, ASA, AAS
  - Use CPCTC (must have congruent triangles!)
- Establishing further relationship
  - ie: Bisect/etc
  - Get triangles congruent
  - Use CPCTC
  - Establish relationship

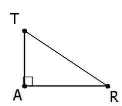
Example: Proof

- Given: $\overline{PR} \cong \overline{AE}$
  $\overline{PE} \cong \overline{AR}$
- Prove: $\overline{PR} \parallel \overline{AE}$



Right Triangles

- The side opposite the right angle is the **hypotenuse** of the triangle.
- The sides of the right angle are the **legs** of the triangle.
- Name the legs of ΔTAR
- Name the hypotenuse of ΔTAR.



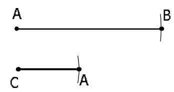
- **Q**: When naming a right triangle, does the right angle always need to be in the middle?

Method for Proving Congruence: HL

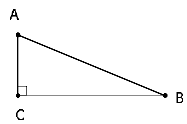
- **Thm 3.2.1**: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent. (HL)

Example: Construction

▸ Given: $\overline{AB}$ and $\overline{CA}$



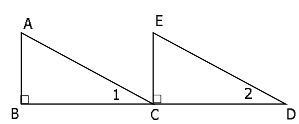
▸ Construct: The right triangle with hypotenuse of length AB and one leg of length CA.



Example

▸ Cite the reason why the right triangles ΔABC and ΔECD are congruent if:

a) AB ≃ EC and AC ≃ ED
b) ∠A ≃ ∠E and C is the midpoint of BD
c) BC ≃ CD and ∠1 ≃ ∠2
d) AB ≃ EC and EC bisects BD



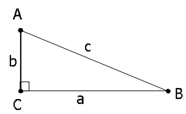
Pythagorean Theorem

▸ The square of the length (c) of the hypotenuse of a right triangle equals the sum of squares of the lengths (a and b) of the legs of the right triangle
  - $c^2 = a^2 + b^2$

▸ **<u>Square Roots Property</u>**: If $x^2 = p > 0$, then $x = \sqrt{p}$.

Example

- Find c if a = 6 and b = 8.
- Find b if a = 7 and c = 10.

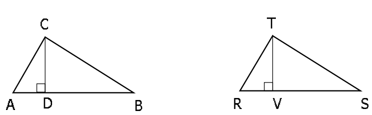


Lines Related to the Triangle

1. **Altitude** –
   - the line segment from a vertex perpendicular to the opposite side.
2. **Angle bisector** –
   - the ray (line segment) from a vertex that bisects that angle.
3. **Perpendicular bisector of a side** –
   - the line that is perpendicular to a side at its midpoint
4. **Median** –
   - the line segment that joins a vertex to the midpoint on the opposite side.

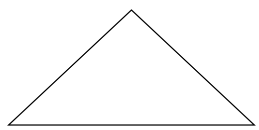
Thm 3.3.1: Corresponding altitudes of congruent triangles are congruent.

- Given: $\Delta ABC \cong \Delta RST$
  Altitudes $\overline{CD}$ to $\overline{AB}$ and $\overline{TV}$ to $\overline{RS}$
- Prove: $\overline{CD} \cong \overline{TV}$



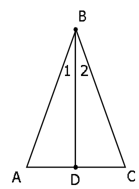
Parts of an Isosceles Triangle

- What makes an isosceles triangle?
- Label: legs, base, vertex, vertex angle, base angles.



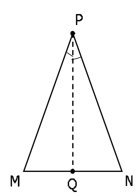
<u>Thm 3.3.2</u>: The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

- Given: Isosceles ΔABC, with $\overline{AB} \cong \overline{BC}$
  $\overrightarrow{BD}$ bisects $\angle ABC$
- Prove: $\Delta ABD \cong \Delta CBD$



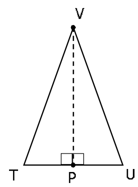
<u>Thm 3.3.3</u>: If two sides of a triangle are congruent, then the angles opposite these sides are also congruent.

- Given: Isosceles ΔMNP, with $\overline{MP} \cong \overline{NP}$
- Prove: $\angle M \cong \angle N$



Thm 3.3.4: If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.

- Given: ΔTUV with $\angle T \cong \angle U$
- Prove: $\overline{VU} \cong \overline{VT}$



Corollaries

- **Cor 3.3.5**: An equilateral triangle is also equiangular.
- **Cor 3.3.6**: An equiangular triangle is also equilateral.

Example

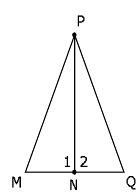
- Complete the statements of theorems:
  1. If two sides of a triangle are congruent, ___.
  2. If two angles of a triangle are congruent, ___.
  3. If a triangle is equilateral, ___.
  4. If a triangle is equiangular, ___.

Example: True/False

- If a triangle is isosceles, at least two of its angles must be equal.
- If a triangle is equiangular, all three of its sides must be equal.
- If a triangle is scalene, none of its angles can be equal.
- If a triangle is equilateral, it is also isosceles.

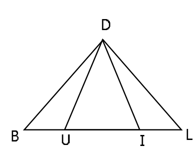
Example

- PE = EA
- "If two sides of a triangle are congruent, then the angles opposite these sides are also congruent."
  a) Can we use this to prove ∠1 ≃ ∠2?
  b) Could you conclude ∠1 ≃ ∠2 if ∠MPN ≃ ∠QPN?



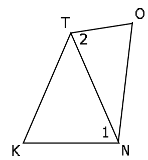
Example

- ∠B ≅ ∠L
- Can you conclude DU = DI?
- Can you conclude DB = DL?



Example: Proof

- Given: ∠1 ≅ ∠K
  ∠O ≅ ∠2
- Prove: KT ≅ ON



Homework

- Due Monday 6/28
  - Read Sections 3.2 and 3.3
  - 3.2: #1-30
  - 3.3: #1-8, 16-22, 27-33, 38-40