

# Math 119 – Plane Geometry

Sections 3.2 and 3.3  
Triangles II  
6/24/2004

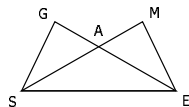
## Recall: Congruence of Triangles

- ▶ **Def:** Two triangles are **congruent** when the six parts of the first triangle are congruent to the six corresponding parts of the second triangle.
- ▶ Methods for Proving Congruence:
  - SSS
  - SAS
  - ASA
  - AAS
- ▶ **CPCTC:** Corresponding parts of congruent triangles are congruent.

## Example

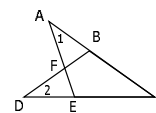
- ▶  $\triangle SGE \cong \triangle EMS$

- a)  $SG = \underline{\hspace{2cm}}$
- b)  $MS = \underline{\hspace{2cm}}$
- c)  $SE = \underline{\hspace{2cm}}$
- d)  $m\angle G = \underline{\hspace{2cm}}$
- e)  $m\angle GSE = \underline{\hspace{2cm}}$
- f)  $m\angle ESM = \underline{\hspace{2cm}}$



## Suggestions: Proving Triangles $\cong$

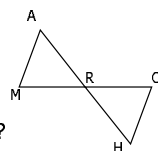
- ▶ Mark figures using the info from your givens
  - Right angles – *squares*
  - Congruent sides – *dashes*
  - Congruent angles – *arcs*
- ▶ Trace triangles to be proved congruent in different colors
- ▶ If triangles overlap, draw them separately



## Example

- ▶  $\angle M$  and  $\angle C$  are supplements of  $\angle ARC$
- ▶  $MR = RC$
- ▶  $\angle ARM$  and  $\angle HRC$  are vertical angles

  - a) Why is  $\angle M \cong \angle C$ ?
  - b) Why is  $\angle ARM \cong \angle HRC$ ?
  - c) Why is  $\triangle MAR \cong \triangle CHR$ ?
  - d) Why is  $AR = RH$ ?
  - e) Why is R the midpoint of  $\overline{AH}$ ?



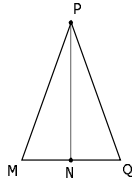
## Example: Proof

- ▶ Given:  $\overline{PN}$  bisects  $\angle MPQ$   
 $\overline{PM} \cong \overline{PQ}$
- ▶ Prove:  $\overline{MN} \cong \overline{QN}$



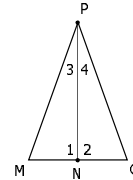
### Example: Proof

- ▶ Given:  $\overline{PN}$  bisects  $\angle MPQ$   
 $\overline{PM} \cong \overline{PQ}$
- ▶ Prove:  $\overline{PN}$  bisects  $\overline{MQ}$



### Example: Proof

- ▶ Given:  $\overline{PM} \cong \overline{PQ}$   
 $\angle 3 \cong \angle 4$   
 $M-N-Q$
- ▶ Prove:  $\overline{PN} \perp \overline{MQ}$

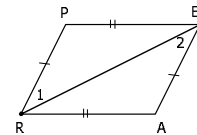


### Triangle Proofs: 3 Types of Conclusions

- ▶ Proving triangles congruent
  - Use SSS, SAS, ASA, AAS
- ▶ Proving corresponding parts of congruent triangles congruent
  - Get triangles congruent using SSS, SAS, ASA, AAS
  - Use CPCTC (must have congruent triangles!)
- ▶ Establishing further relationship
  - ie: Bisect/etc
  - Get triangles congruent
  - Use CPCTC
  - Establish relationship

### Example: Proof

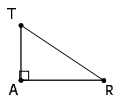
- ▶ Given:  $\overline{PR} \cong \overline{AE}$   
 $\overline{PE} \cong \overline{AR}$
- ▶ Prove:  $\overline{PR} \parallel \overline{AE}$



### Right Triangles

- ▶ The side opposite the right angle is the **hypotenuse** of the triangle.
- ▶ The sides of the right angle are the **legs** of the triangle.

- ▶ Name the legs of  $\triangle TAR$
- ▶ Name the hypotenuse of  $\triangle TAR$ .



- ▶ **Q:** When naming a right triangle, does the right angle always need to be in the middle?

### Method for Proving Congruence: HL

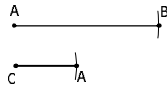
- ▶ **Thm 3.2.1:** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent. (HL)



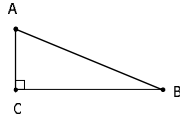
### Example: Construction

► Given:

$\overline{AB}$  and  $\overline{CA}$

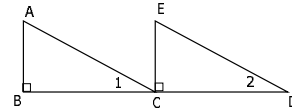


► Construct: The right triangle with hypotenuse of length AB and one leg of length CA.



### Example

- Cite the reason why the right triangles  $\triangle ABC$  and  $\triangle ECD$  are congruent if:
- $AB \cong EC$  and  $AC \cong ED$
  - $\angle A \cong \angle E$  and C is the midpoint of BD
  - $BC \cong CD$  and  $\angle 1 \cong \angle 2$
  - $AB \cong EC$  and EC bisects BD



### Pythagorean Theorem

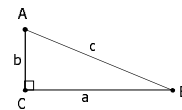
► The square of the length (c) of the hypotenuse of a right triangle equals the sum of squares of the lengths (a and b) of the legs of the right triangle

▪  $c^2 = a^2 + b^2$

► **Square Roots Property:** If  $x^2 = p > 0$ , then  $x = \sqrt{p}$ .

### Example

- Find c if a = 6 and b = 8.
- Find b if a = 7 and c = 10.

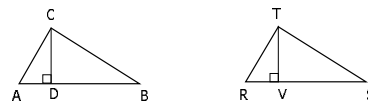


### Lines Related to the Triangle

- Altitude** –
  - the line segment from a vertex perpendicular to the opposite side.
- Angle bisector** –
  - the ray (line segment) from a vertex that bisects that angle.
- Perpendicular bisector of a side** –
  - the line that is perpendicular to a side at its midpoint
- Median** –
  - the line segment that joins a vertex to the midpoint on the opposite side.

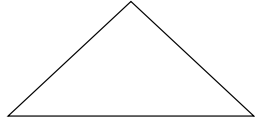
**Thm 3.3.1:** Corresponding altitudes of congruent triangles are congruent.

- Given:  $\triangle ABC \cong \triangle RST$   
 Altitudes  $\overline{CD}$  to  $\overline{AB}$  and  $\overline{TV}$  to  $\overline{RS}$
- Prove:  $\overline{CD} \cong \overline{TV}$



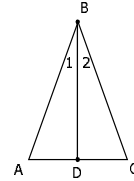
## Parts of an Isosceles Triangle

- ▶ What makes an isosceles triangle?
- ▶ Label: legs, base, vertex, vertex angle, base angles.



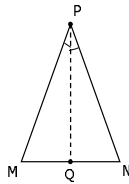
**Thm 3.3.2:** The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

- ▶ Given: Isosceles  $\triangle ABC$ , with  $\overline{AB} \cong \overline{BC}$   
 $\overrightarrow{BD}$  bisects  $\angle ABC$
- ▶ Prove:  $\triangle ABD \cong \triangle CBD$



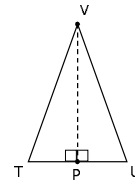
**Thm 3.3.3:** If two sides of a triangle are congruent, then the angles opposite these sides are also congruent.

- ▶ Given: Isosceles  $\triangle MNP$ , with  $\overline{MP} \cong \overline{NP}$
- ▶ Prove:  $\angle M \cong \angle N$



**Thm 3.3.4:** If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.

- ▶ Given:  $\triangle TUV$  with  $\angle T \cong \angle U$
- ▶ Prove:  $\overline{VU} \cong \overline{VT}$



## Corollaries

- ▶ **Cor 3.3.5:** An equilateral triangle is also equiangular.
- ▶ **Cor 3.3.6:** An equiangular triangle is also equilateral.

## Example

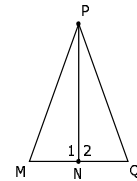
- ▶ Complete the statements of theorems:
  1. If two sides of a triangle are congruent, \_\_\_\_.
  2. If two angles of a triangle are congruent, \_\_\_\_.
  3. If a triangle is equilateral, \_\_\_\_.
  4. If a triangle is equiangular, \_\_\_\_.

### Example: True/False

- ▶ If a triangle is isosceles, at least two of its angles must be equal.
- ▶ If a triangle is equiangular, all three of its sides must be equal.
- ▶ If a triangle is scalene, none of its angles can be equal.
- ▶ If a triangle is equilateral, it is also isosceles.

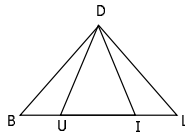
### Example

- ▶  $PE = EA$
- ▶ "If two sides of a triangle are congruent, then the angles opposite these sides are also congruent."
  - Can we use this to prove  $\angle 1 \cong \angle 2$ ?
  - Could you conclude  $\angle 1 \cong \angle 2$  if  $\angle MPN \cong \angle QPN$ ?



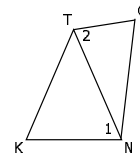
### Example

- ▶  $\angle B \cong \angle L$
- ▶ Can you conclude  $DU = DI$ ?
- ▶ Can you conclude  $DB = DL$ ?



### Example: Proof

- ▶ Given:  $\angle 1 \cong \angle K$   
 $\angle O \cong \angle 2$
- ▶ Prove:  $KT \cong ON$



### Homework

- ▶ Due Monday 6/28
  - Read Sections 3.2 and 3.3
  - 3.2: #1-30
  - 3.3: #1-8, 16-22, 27-33, 38-40