

Math 119 – Plane Geometry

Sections 4.3 and 4.4
Quadrilaterals II
6/30/2004

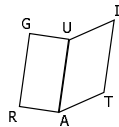
Warm Up Example

► Complete the following statements:

1. The opposite sides of a parallelogram are ___ and ___.
2. The consecutive angles of a parallelogram are ___.
3. The opposite angles of a parallelogram are ___.
4. The diagonals of a parallelogram are ___.

Warm Up Example

- Given: \overline{GUAR} and \overline{UITA} are parallelograms
► Prove: $\overline{GR} \cong \overline{IT}$



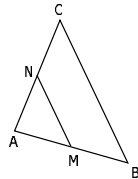
Warm Up Example: True/False

1. If a quadrilateral is a parallelogram, then at least two opposite sides are parallel.
2. A quadrilateral is a parallelogram if at least two opposite sides are parallel.
3. If a quadrilateral is a parallelogram, then its diagonals bisect each other.
4. If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.
5. If a quadrilateral is a parallelogram, then it is equilateral.
6. A quadrilateral is a parallelogram if it is equilateral.

Warm Up Example

► MN is a midsegment of $\triangle ABC$.
 $m\angle C = m\angle B = 60$. $CB = 16$.

1. Find $m\angle A$.
2. What kind of triangle is $\triangle ABC$?
3. What kind of triangle is $\triangle AMN$?
4. What kind of quadrilateral is $MNCB$?
5. Find MN.
6. Find RA.



Recall: Types of Parallelograms

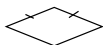
► A **rectangle** is a parallelogram that has ____.



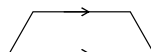
► A **square** is a rectangle that has ____.



► A **rhombus** is a parallelogram with ____.



► A **trapezoid** is a quadrilateral with ____.



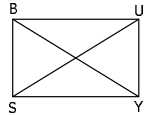
Theorems About Rectangles

► **Cor 4.3.1:** All angles of a rectangle are right angles.

► **Thm 4.3.2:** The diagonals of a rectangle are congruent.

▪ Given: Rectangle BUSY with diagonals \overline{SU} and \overline{BY}

▪ Prove: $\overline{SU} \cong \overline{BY}$



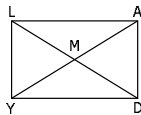
▪ Plan: Examine $\triangle SBU$ and $\triangle YUB$

Example

► Quadrilateral LADY is a rectangle. \overline{LD} and \overline{AY} are its diagonals.

1. What kind of triangle is $\triangle ADY$? Why?

2. What kind of triangle is $\triangle YMD$? Why?

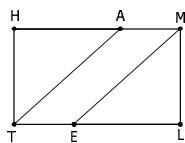


Example

► Given: HMLT is a rectangle

$\overline{HA} \cong \overline{EL}$

► Prove: $\overline{TA} \cong \overline{EM}$

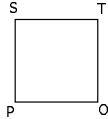


Theorems About Squares

► **Cor 4.3.3:** All sides of a square are congruent.

► **Ex:** Quadrilateral STOP is a square.

1. Why is STOP a rhombus?
2. Why is STOP a parallelogram?
3. Why is STOP a rectangle?



Theorems About Rhombuses

► **Cor 4.3.4:** All sides of a rhombus are congruent.

► **Thm 4.3.5:** The diagonals of a rhombus are perpendicular.

- Given: Rhombus $ABCD$ with diagonals \overline{AC} and \overline{DB}
- Prove: $\overline{AC} \perp \overline{DB}$
- Plan: Examine $\triangle ABE$ and $\triangle CBE$



Example

► Quadrilateral $ABCD$ is a rhombus and \overline{BD} and \overline{AC} are its diagonals.

1. What kind of triangle is $\triangle ABC$?
2. What kind of triangle is $\triangle ABE$?



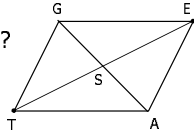
Example

► In quadrilateral GATE, \overline{GA} and \overline{ET} bisect each other.

1. What kind of quadrilateral is GATE? Why?

2. Does it follow that $\overline{GA} \perp \overline{ET}$?

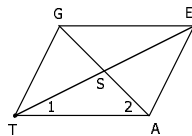
3. Does it follow that $\overline{GA} \cong \overline{ET}$?



Example

► Given: GATE is a rhombus with diagonals \overline{GA} and \overline{ET}

► Prove: $\angle 1$ and $\angle 2$ are complementary



Pythagorean Theorem Revisited

► In a right triangle with hypotenuse of length c and legs of lengths a and b , $c^2 = a^2 + b^2$.

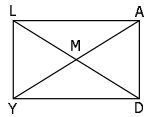
► **Ex:** What is the length of the diagonal in a rectangle whose sides measure 3 ft and 4 ft?

► **Ex:** What is the length of each side of a rhombus whose diagonals measure 10 cm and 24 cm?

Example

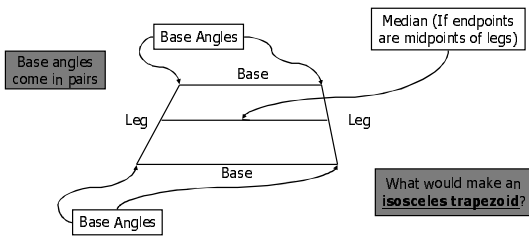
- ▶ Quadrilateral $LADY$ is a rectangle with diagonals \overline{LD} and \overline{AY} .
- ▶ $LY = 10$ and $YD = 24$

1. Find LD .
2. Find LM .



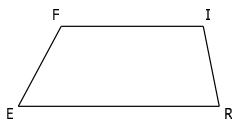
The Trapezoid

- ▶ A **trapezoid** is a quadrilateral with exactly two parallel sides.



Example

- ▶ Quadrilateral $FIRE$ is a trapezoid with $\overline{FI} \parallel \overline{ER}$
1. Which sides are the bases?
 2. Which sides are the legs?
 3. Does $FIRE$ appear to be isosceles?
- ▶ Suppose $m\angle F = 107$ and $m\angle R = 58$. Find the measures of the other angles.

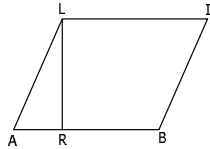


Example: True/False

1. If a quadrilateral is a trapezoid, its base angles are equal.
2. A trapezoid can have three equal sides.
3. A trapezoid can have four equal angles.
4. A trapezoid can have three right angles.
5. If the diagonals of a quadrilateral bisect each other, it is not a trapezoid.

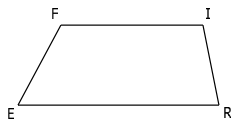
Example

- Given: $LIBR$ is a trapezoid with bases \overline{LI} and \overline{RB}
 $\overline{LI} \cong \overline{BA}$
- Prove: $LIBA$ is a parallelogram



Altitudes of Trapezoids

- An **altitude** of a trapezoid is a line segment from one vertex of one base of the trapezoid perpendicular to the opposite base (or to an extension of that base).
- **Ex:** Draw altitudes from each of the four vertices:



Isosceles Trapezoid Theorems

► **Thm 4.4.1:** The base angles of an isosceles trapezoid are congruent.

▪ Given: Trapezoid SILK

$$\overline{SK} \cong \overline{IL} \text{ and } \overline{SI} \parallel \overline{KL}$$

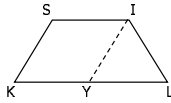
▪ Prove: $\angle S \cong \angle I$ and $\angle K \cong \angle L$

▪ Plan:

► Construct Y so that $\overline{IY} \parallel \overline{SK}$

► Get $\angle K \cong \angle L$

► Then get $\angle S \cong \angle I$



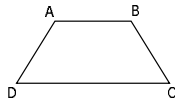
► **Cor 4.4.2:** The diagonals of an isosceles trapezoid are congruent. (SAS; cpctc)

Example

► Given isosceles trapezoid ABCD with $\overline{AB} \parallel \overline{DC}$

1. Find the measures of the angles of ABCD if $m\angle A = 12x + 30$ and $m\angle B = 10x + 46$.

2. Using the result from part (1), find the length of each diagonal if $AC = 2x - 5$.



Theorems For Medians of Trapezoids

► **Thm 4.4.3:** The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.

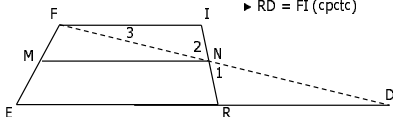
► **Thm 4.4.4:** The median of a trapezoid is parallel to each base.

► Given: Trapezoid FIRE
median \overline{MN}
 $\overline{FI} \parallel \overline{ER}$

► Prove: $\overline{MN} \parallel \overline{ER}$ and
 $MN = \frac{1}{2} (FI + ER)$

► Plan

- Construct line \overline{FN}
- $\triangle DNR \cong \triangle FNI$ by AAS
- Get \overline{MN} as midsegment of $\triangle EFD$
 - 4.4.3 Follows
 - $MN = \frac{1}{2} (ED) = \frac{1}{2} (ER + RD)$
 - $RD = FI$ (cpctc)

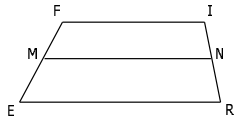


Example

► In trapezoid $FIRE$, $\overline{FI} \parallel \overline{ER}$ and M and N are midpoints of \overline{FE} and \overline{IR} , respectively.

1. Find the length of median \overline{MN} if $FI = 12$ and $ER = 18$.

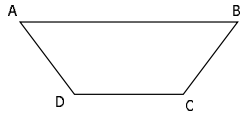
2. Find MN , FI , and ER if $FI = 2x$, $MN = 3x - 5$, and $ER = 2x + 10$.



Proving Quadrilaterals Trapezoids

► **Thm 4.4.5:** If two of three consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid.

- What kind of a figure would it be if 2 of 2 consecutive angles were supplementary?



Proving Isosceles Trapezoids

► **Thm 4.4.6:** If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

► **Thm 4.4.7:** If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

Transversals

► **Thm 4.4.8:** If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.

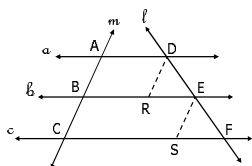
► **Given:** Parallel lines a , b , and c

$$\overline{AB} \cong \overline{BC}$$

► **Prove:** $\overline{DE} \cong \overline{EF}$

► **Plan:**

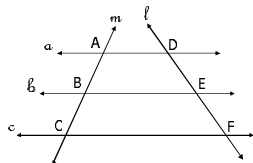
- Draw \overline{DR} and \overline{ES} parallel to ℓ
- Show $\triangle DER \cong \triangle EFS$ by AAS



Example

► **Given:** Parallel lines a , b , and c .

► If $AB = BC = 7.2$ and $DE = 8.4$, find EF .



Homework

► **Due Thursday 7/1**

- Read Sections 4.3 and 4.4
- 4.3: #1-31
- 4.4: #1-18, 21-23, 25, 26

► **Reminder:** Exam 3 – Tuesday, July 6
