

# Math 119 – Plane Geometry

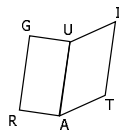
Sections 4.3 and 4.4  
Quadrilaterals II  
6/30/2004

## Warm Up Example

- Complete the following statements:
1. The opposite sides of a parallelogram are \_\_\_ and \_\_\_.
  2. The consecutive angles of a parallelogram are \_\_\_.
  3. The opposite angles of a parallelogram are \_\_\_.
  4. The diagonals of a parallelogram are \_\_\_.

## Warm Up Example

- Given:  $\overline{GUAR}$  and  $\overline{UITA}$  are parallelograms  
► Prove:  $\overline{GR} \cong \overline{IT}$



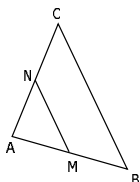
## Warm Up Example: True/False

1. If a quadrilateral is a parallelogram, then at least two opposite sides are parallel.
2. A quadrilateral is a parallelogram if at least two opposite sides are parallel.
3. If a quadrilateral is a parallelogram, then its diagonals bisect each other.
4. If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.
5. If a quadrilateral is a parallelogram, then it is equilateral.
6. A quadrilateral is a parallelogram if it is equilateral.

### Warm Up Example

► MN is a midsegment of  $\triangle ABC$ ,  
 $m\angle C = m\angle B = 60$ ,  $CB = 16$ .

1. Find  $m\angle A$ .
2. What kind of triangle is  $\triangle ABC$ ?
3. What kind of triangle is  $\triangle AMN$ ?
4. What kind of quadrilateral is  $MNCB$ ?
5. Find MN.
6. Find RA.



### Recall: Types of Parallelograms

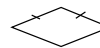
► A **rectangle** is a parallelogram that has \_\_\_\_.



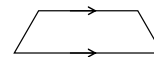
► A **square** is a rectangle that has \_\_\_\_.



► A **rhombus** is a parallelogram with \_\_\_\_.



► A **trapezoid** is a quadrilateral with \_\_\_\_.

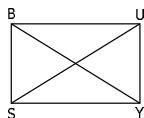


### Theorems About Rectangles

► **Cor 4.3.1:** All angles of a rectangle are right angles.

► **Thm 4.3.2:** The diagonals of a rectangle are congruent.

- Given: Rectangle BUSY with diagonals  $\overline{SU}$  and  $\overline{BY}$
- Prove:  $SU \cong BY$

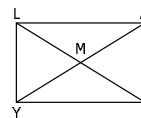


- Plan: Examine  $\triangle SBU$  and  $\triangle YUB$

### Example

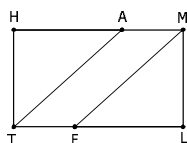
► Quadrilateral LADY is a rectangle.  $\overline{LD}$  and  $\overline{AY}$  are its diagonals.

1. What kind of triangle is  $\triangle ADY$ ? Why?
2. What kind of triangle is  $\triangle YMD$ ? Why?



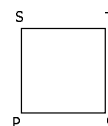
### Example

- ▶ Given:  $HMLT$  is a rectangle  
 $\overline{HA} \cong \overline{EL}$
- ▶ Prove:  $\overline{TA} \cong \overline{EM}$



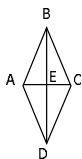
### Theorems About Squares

- ▶ **Cor 4.3.3:** All sides of a square are congruent.
- ▶ **Ex:** Quadrilateral STOP is a square.
  1. Why is STOP a rhombus?
  2. Why is STOP a parallelogram?
  3. Why is STOP a rectangle?



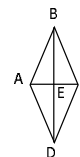
### Theorems About Rhombuses

- ▶ **Cor 4.3.4:** All sides of a rhombus are congruent.
- ▶ **Thm 4.3.5:** The diagonals of a rhombus are perpendicular.
  - Given: Rhombus  $ABCD$  with diagonals  $\overline{AC}$  and  $\overline{DB}$
  - Prove:  $\overline{AC} \perp \overline{DB}$
  - Plan: Examine  $\triangle ABE$  and  $\triangle CBE$



### Example

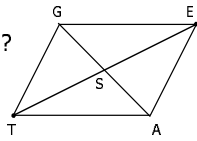
- ▶ Quadrilateral  $ABCD$  is a rhombus and  $\overline{BD}$  and  $\overline{AC}$  are its diagonals.
  1. What kind of triangle is  $\triangle ABC$ ?
  2. What kind of triangle is  $\triangle ABE$ ?



### Example

► In quadrilateral  $GATE$ ,  $\overline{GA}$  and  $\overline{ET}$  bisect each other.

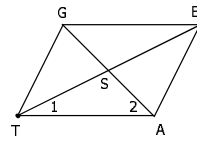
1. What kind of quadrilateral is  $GATE$ ? Why?
2. Does it follow that  $\overline{GA} \perp \overline{ET}$ ?
3. Does it follow that  $\overline{GA} \cong \overline{ET}$ ?



### Example

► Given:  $GATE$  is a rhombus with diagonals  $\overline{GA}$  and  $\overline{ET}$

► Prove:  $\angle 1$  and  $\angle 2$  are complementary



### Pythagorean Theorem Revisited

► In a right triangle with hypotenuse of length  $c$  and legs of lengths  $a$  and  $b$ ,  $c^2 = a^2 + b^2$ .

► **Ex:** What is the length of the diagonal in a rectangle whose sides measure 3 ft and 4 ft?

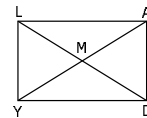
► **Ex:** What is the length of each side of a rhombus whose diagonals measure 10 cm and 24 cm?

### Example

► Quadrilateral  $LADY$  is a rectangle with diagonals  $\overline{LD}$  and  $\overline{AY}$ .

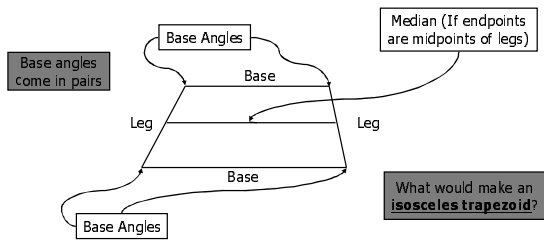
►  $LY = 10$  and  $YD = 24$

1. Find  $LD$ .
2. Find  $LM$ .



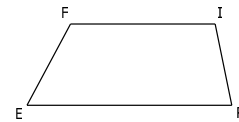
## The Trapezoid

- A **trapezoid** is a quadrilateral with exactly two parallel sides.



## Example

- Quadrilateral FIRE is a trapezoid with  $\overline{FI} \parallel \overline{ER}$
1. Which sides are the bases?
  2. Which sides are the legs?
  3. Does FIRE appear to be isosceles?
- Suppose  $m\angle F = 107$  and  $m\angle R = 58$ . Find the measures of the other angles.

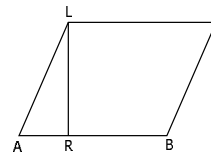


## Example: True/False

1. If a quadrilateral is a trapezoid, its base angles are equal.
2. A trapezoid can have three equal sides.
3. A trapezoid can have four equal angles.
4. A trapezoid can have three right angles.
5. If the diagonals of a quadrilateral bisect each other, it is not a trapezoid.

## Example

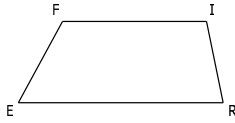
- Given: LIBR is a trapezoid with bases  $\overline{LI}$  and  $\overline{RB}$   
 $\overline{LI} \cong \overline{BA}$
- Prove: LIBA is a parallelogram



## Altitudes of Trapezoids

► An **altitude** of a trapezoid is a line segment from one vertex of one base of the trapezoid perpendicular to the opposite base (or to an extension of that base).

► **Ex:** Draw altitudes from each of the four vertices:



## Isosceles Trapezoid Theorems

► **Thm 4.4.1:** The base angles of an isosceles trapezoid are congruent.

▪ Given: Trapezoid SILK

$$\overline{SK} \cong \overline{IL} \text{ and } \overline{SI} \parallel \overline{KL}$$

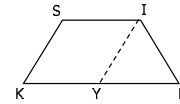
▪ Prove:  $\angle S \cong \angle I$  and  $\angle K \cong \angle L$

▪ Plan:

► Construct Y so that  $\overline{IY} \parallel \overline{SK}$

► Get  $\angle K \cong \angle L$

► Then get  $\angle S \cong \angle I$

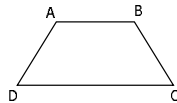


► **Cor 4.4.2:** The diagonals of an isosceles trapezoid are congruent. (SAS; cpctc)

## Example

► Given isosceles trapezoid ABCD with  $\overline{AB} \parallel \overline{DC}$

- Find the measures of the angles of ABCD if  $m\angle A = 12x + 30$  and  $m\angle B = 10x + 46$ .
- Using the result from part (1), find the length of each diagonal if  $AC = 2x - 5$ .



## Theorems For Medians of Trapezoids

► **Thm 4.4.3:** The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.

► **Thm 4.4.4:** The median of a trapezoid is parallel to each base.

► **Given:** Trapezoid FIRE  
median  $\overline{MN}$   
 $\overline{FI} \parallel \overline{ER}$

► **Prove:**  $\overline{MN} \parallel \overline{ER}$  and  
 $MN = \frac{1}{2}(FI + ER)$

► Plan

▪ Construct line  $\overline{FN}$

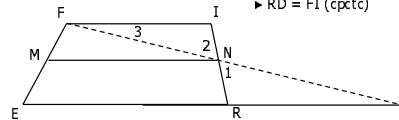
▪  $\triangle DNR \cong \triangle FNI$  by AAS

▪ Get  $\overline{MN}$  as midsegment of  $\triangle EFD$

► 4.4.3 Follows

►  $MN = \frac{1}{2}(ED) = \frac{1}{2}(ER + RD)$

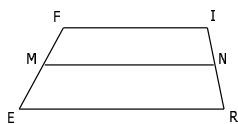
►  $RD = FI$  (cpctc)



### Example

► In trapezoid FIRE,  $\overline{FI} \parallel \overline{ER}$  and M and N are midpoints of  $\overline{FE}$  and  $\overline{IR}$ , respectively.

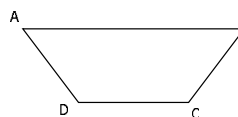
1. Find the length of median  $\overline{MN}$  if  $FI = 12$  and  $ER = 18$ .
2. Find MN, FI, and ER if  $FI = 2x$ ,  $MN = 3x - 5$ , and  $ER = 2x + 10$ .



### Proving Quadrilaterals Trapezoids

► **Thm 4.4.5:** If two of three consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid.

- What kind of a figure would it be if 2 of 2 consecutive angles were supplementary?



### Proving Isosceles Trapezoids

► **Thm 4.4.6:** If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

► **Thm 4.4.7:** If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.

### Transversals

► **Thm 4.4.8:** If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.

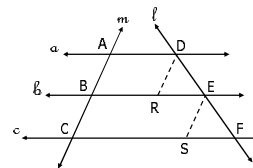
► Given: Parallel lines  $a$ ,  $b$ , and  $c$

$$\overline{AB} \cong \overline{BC}$$

► Prove:  $\overline{DE} \cong \overline{EF}$

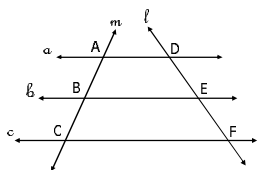
► Plan:

- Draw  $\overline{DR}$  and  $\overline{ES}$  parallel to  $\ell$
- Show  $\triangle DER \cong \triangle EFS$  by AAS



### Example

- ▶ Given: Parallel lines  $a$ ,  $b$ , and  $c$ .
- ▶ If  $AB = BC = 7.2$  and  $DE = 8.4$ , find  $EF$ .



### Homework

- ▶ Due Thursday 7/1
  - Read Sections 4.3 and 4.4
  - 4.3: #1-31
  - 4.4: #1-18, 21-23, 25, 26
- ▶ Reminder: Exam 3 – Tuesday, July 6