Math 119 – Plane Geometry

Sections 5.2 and 5.3
Similarity II
7/6/2004

Recap of Similarity

- Two polygons are **similar** if, and only if,
  - All pairs of corresponding angles are congruent
  - All pairs of corresponding sides are proportional
- **AA Similarity Theorem**: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
- **CSSTP**: Corresponding sides of similar triangles are proportional

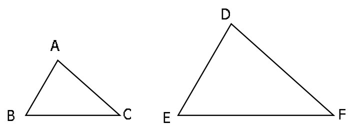
Other Ways To Prove Similarity

- **SAS~**: If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including the angles are proportional, then the triangles are similar.
- **SSS~**: If the three sides of one triangle are proportional to the three corresponding sides of a second triangle, then the triangles are similar.

Example

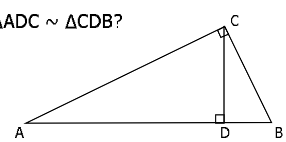
- Which method (AA~, SAS~, or SSS~) establishes that ΔABC ~ ΔDEF?

1. ∠A ≅ ∠D, AC = 6, DF = 9, AB = 8, and DE = 12
2. AB = 6, AC = 4, BC = 8, DE = 9, DF = 6, and EF = 12



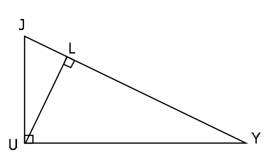
Similarities in Right Triangles

- **<u>Thm 5.3.1</u>**: The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original right triangle. (HW)
  - Why are ΔADC ~ ΔACB and ΔCDB ~ ΔACB?
  - Why does this mean ΔADC ~ ΔCDB?



Example

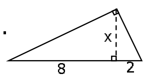
1. ΔJUL ~ ΔJ___
2. ΔYUJ ~ ΔY___
3. ΔLJU ~ ΔL___
4. JY/JU = JU/____
5. JY/UY = UY/____
6. JL/LU = LU/____



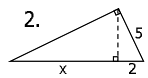
Example

- Solve for x:

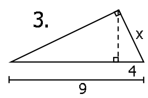
1.



2.



3.

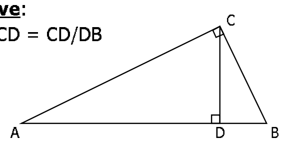


Two More Results

- **<u>Thm 5.3.2</u>**: The length of the altitude of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.
- **<u>Thm 5.3.3</u>**: The length of each leg of a right triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.

<u>Given</u>: ΔABC with right ∠ACB, $\overline{CD} \perp \overline{AB}$

<u>Prove</u>:
AD/CD = CD/DB

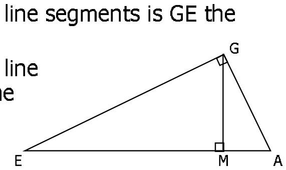


<u>Prove</u>:
AB/AC = AC/AD

Use CSSTP

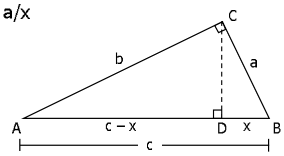
Example

1. What is EA called with respect to ΔGEA?
2. What is GM called?
3. Between which two segments in the figure is GM the geometric mean?
4. What are GE and GA called with respect to ΔGEA?
5. Between which two line segments is GE the geometric mean?
6. Between which two line segments is GA the geometric mean?



Once More: Pythagorean Thm

- **Pythagorean Thm**: The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.
- Given: ΔABC with right ∠C
- Prove: $a^2 + b^2 = c^2$
- Steps:
  - Construct $\overline{CD} \perp \overline{DB}$
  - $c/b = b/(c - x)$; $c/a = a/x$
  - $b^2 = c^2 - cx$; $a^2 = cx$
  - Add the equations…

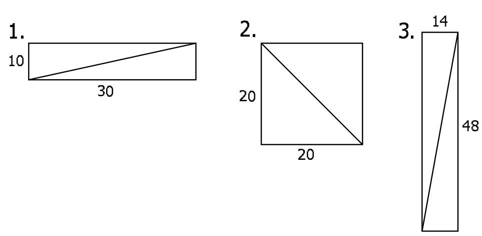


Example

- One diagonal of a rhombus has the same length, 10 cm, as each side. How long is the other diagonal?
  - Why do the diagonals bisect each other?
  - Why can we apply the Pythagorean Theorem?

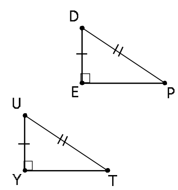
Example

- Find the length of the diagonal in each of these rectangles:



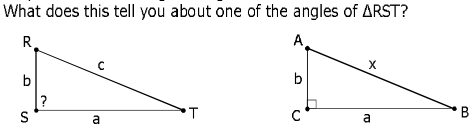
Proof of HL Congruence Theorem

- **Thm 5.3.6**: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.
  - Why is EP = YT?
  - Why can we then conclude the triangles are congruent?



Converse of Pythagorean Thm

- **Converse of Pyhtagorean Thm**: If a, b, and c are the lengths of the three sides of a triangle, with c the length of the longest side, and if $c^2 = a^2 + b^2$, then the triangle is a right triangle with right angle opposite the side of length c.
- Given: ΔRST with $a^2 + b^2 = c^2$
- Prove: ΔRST is a right triangle
  - Construct ΔABC with legs of length a and b and hypotenuse of length x.
  - What does the Pythagorean Thm tell us about x?
  - Why are the two triangles congruent?
  - What does this tell you about one of the angles of ΔRST?

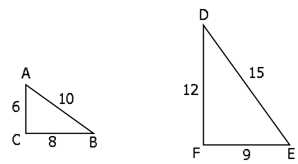


Example

- Which of the following can be lengths of the sides of a right triangle?
  1. 5, 12, 13
  2. 15, 8, 17
  3. 7, 9, 10
  4. $\sqrt{2}, \sqrt{3}, \sqrt{5}$

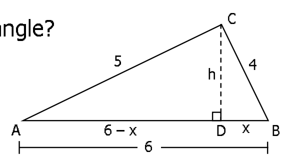
Example

1. Why are angles C and F right angles?
2. Are the following triangles similar?



Example

- ► A triangle has sides of length 4, 5 and 6. Find the length of the altitude to the side of length 6.
  - Apply Pythagorean Theorem to the two triangles formed.
  - Is ΔABC a right triangle?

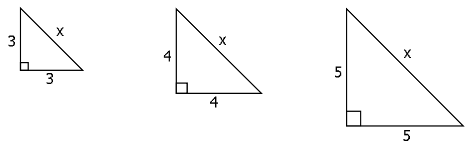


From the Converse:

- ► **Thm 5.3.7**: Let a, b, and c represent the lengths of the three sides of a triangle, with c the length of the longest side.
  - If $c^2 > a^2 + b^2$, then the triangle is obtuse and the obtuse angle lies opposite the side of length c.
  - If $c^2 < a^2 + b^2$, then the triangle is acute.
- ► **Ex:** Determine the type of triangle represented if the lengths of its sides are as follows:
  1. 4, 5, 7
  2. 6, 7, 8
  3. 9, 12, 15
  4. 3, 4, 9

Preview for Next Time

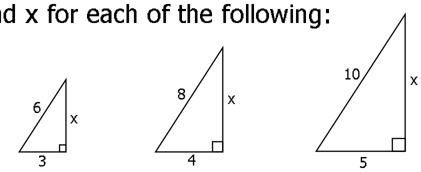
- Find x for each of the following:



- How does the length of the hypotenuse of an isosceles triangle seem to be related to the length of each leg?

Preview for Next Time

- Find x for each of the following:



- If the hypotenuse of a right triangle is twice the length of the shorter leg, how does the length of the longer leg seem to be related to the length of the shorter leg?

Homework

- Due Wednesday 7/7
  - Read Sections 5.2 and 5.3
  - 5.2: #5-12, 20, 41
  - 5.3: #1-10, 15-27, 28 (hint: use indirect proof), 30, 31