

Math 119 – Plane Geometry

Sections 5.2 and 5.3
Similarity II
7/6/2004

Recap of Similarity

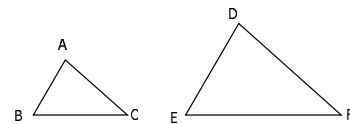
- ▶ Two polygons are **similar** if, and only if,
 - All pairs of corresponding angles are congruent
 - All pairs of corresponding sides are proportional
- ▶ **AA Similarity Theorem:** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
- ▶ **CSSTP:** Corresponding sides of similar triangles are proportional

Other Ways To Prove Similarity

- ▶ **SAS~:** If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including the angles are proportional, then the triangles are similar.
- ▶ **SSS~:** If the three sides of one triangle are proportional to the three corresponding sides of a second triangle, then the triangles are similar.

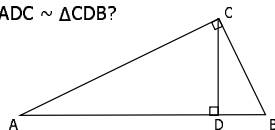
Example

- ▶ Which method (AA~, SAS~, or SSS~) establishes that $\triangle ABC \sim \triangle DEF$?
 1. $\angle A \cong \angle D$, $AC = 6$, $DF = 9$, $AB = 8$, and $DE = 12$
 2. $AB = 6$, $AC = 4$, $BC = 8$, $DE = 9$, $DF = 6$, and $EF = 12$



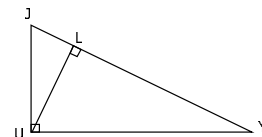
Similarities in Right Triangles

- ▶ **Thm 5.3.1:** The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original right triangle. (HW)
 - Why are $\triangle ADC \sim \triangle ACB$ and $\triangle CDB \sim \triangle ACB$?
 - Why does this mean $\triangle ADC \sim \triangle CDB$?



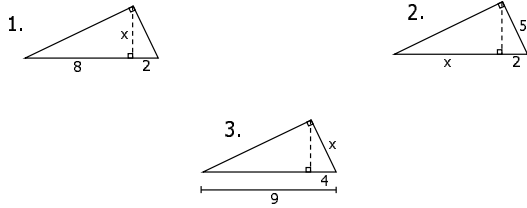
Example

1. $\triangle JUL \sim \triangle J$ ___
2. $\triangle YUJ \sim \triangle Y$ ___
3. $\triangle LJU \sim \triangle L$ ___
4. $JY/JU = JU/$ ___
5. $JY/UY = UY/$ ___
6. $JL/LU = LU/$ ___



Example

► Solve for x:



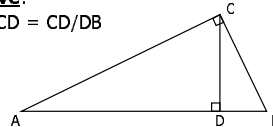
Two More Results

- **Thm 5.3.2:** The length of the altitude of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.
- **Thm 5.3.3:** The length of each leg of a right triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.

Given: $\triangle ABC$ with right $\angle ACB$, $\overline{CD} \perp \overline{AB}$

Prove:
 $AD/CD = CD/DB$

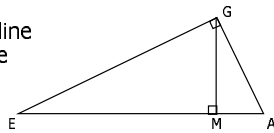
Prove:
 $AB/AC = AC/AD$



Use CSSTP

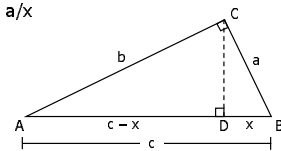
Example

1. What is EA called with respect to $\triangle GEA$?
2. What is GM called?
3. Between which two segments in the figure is GM the geometric mean?
4. What are GE and GA called with respect to $\triangle GEA$?
5. Between which two line segments is GE the geometric mean?
6. Between which two line segments is GA the geometric mean?



Once More: Pythagorean Thm

- **Pythagorean Thm:** The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.
- **Given:** $\triangle ABC$ with right $\angle C$
- **Prove:** $a^2 + b^2 = c^2$
- **Steps:**
 - Construct $\overline{CD} \perp \overline{DB}$
 - $c/b = b/(c-x)$; $c/a = a/x$
 - $b^2 = c^2 - cx$; $a^2 = cx$
 - Add the equations...

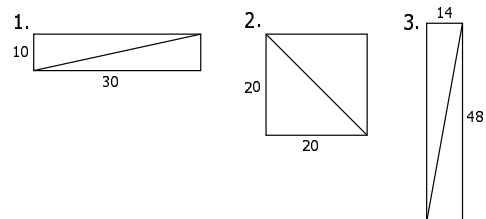


Example

- One diagonal of a rhombus has the same length, 10 cm, as each side. How long is the other diagonal?
- Why do the diagonals bisect each other?
- Why can we apply the Pythagorean Theorem?

Example

- Find the length of the diagonal in each of these rectangles:

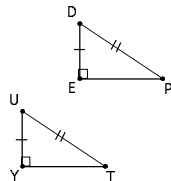


Proof of HL Congruence Theorem

- **Thm 5.3.6:** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

▪ Why is $EP = YT$?

▪ Why can we then conclude the triangles are congruent?



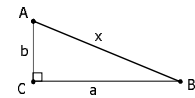
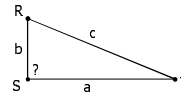
Converse of Pythagorean Thm

- **Converse of Pythagorean Thm:** If a , b , and c are the lengths of the three sides of a triangle, with c the length of the longest side, and if $c^2 = a^2 + b^2$, then the triangle is a right triangle with right angle opposite the side of length c .

► **Given:** ΔRST with $a^2 + b^2 = c^2$

► **Prove:** ΔRST is a right triangle

- Construct ΔABC with legs of length a and b and hypotenuse of length x .
- What does the Pythagorean Thm tell us about x ?
- Why are the two triangles congruent?
- What does this tell you about one of the angles of ΔRST ?



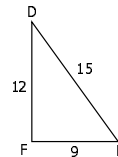
Example

- Which of the following can be lengths of the sides of a right triangle?

- 5, 12, 13
- 15, 8, 17
- 7, 9, 10
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

Example

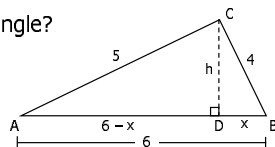
- Why are angles C and F right angles?
- Are the following triangles similar?



Example

- A triangle has sides of length 4, 5 and 6. Find the length of the altitude to the side of length 6.

- Apply Pythagorean Theorem to the two triangles formed.
- Is ΔABC a right triangle?



From the Converse:

- **Thm 5.3.7:** Let a , b , and c represent the lengths of the three sides of a triangle, with c the length of the longest side.

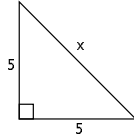
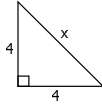
- If $c^2 > a^2 + b^2$, then the triangle is obtuse and the obtuse angle lies opposite the side of length c .
- If $c^2 < a^2 + b^2$, then the triangle is acute.

- **Ex:** Determine the type of triangle represented if the lengths of its sides are as follows:

- 4, 5, 7
- 2.6, 7, 8
- 3.9, 12, 15
- 4.3, 4, 9

Preview for Next Time

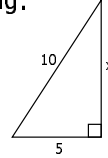
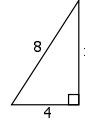
► Find x for each of the following:



► How does the length of the hypotenuse of an isosceles triangle seem to be related to the length of each leg?

Preview for Next Time

► Find x for each of the following:



► If the hypotenuse of a right triangle is twice the length of the shorter leg, how does the length of the longer leg seem to be related to the length of the shorter leg?

Homework

► Due Wednesday 7/7

- Read Sections 5.2 and 5.3
- 5.2: #5-12, 20, 41
- 5.3: #1-10, 15-27, 28 (hint: use indirect proof), 30, 31