

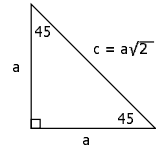
Math 119 – Plane Geometry

Sections 5.4 and 5.5
Similarity III
7/7/2004

45-45-90 Theorem

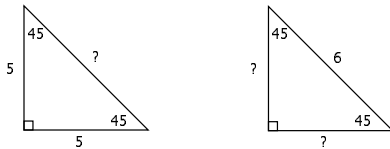
► **45-45-90 Theorem:** In a triangle whose angles measure 45° , 45° , and 90° , the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

- **Given:** Isosceles right triangle with legs of measure a and hypotenuse of measure c
- **Prove:** $c = a\sqrt{2}$
- **Proof:** By Pythagorean Theorem



Examples

► Find the missing lengths:



► A square has a diagonal of length $2\sqrt{2}$. Find the length of its sides.

Converse of 45-45-90 Theorem

► **Thm 5.4.3:** If the length of the hypotenuse of a right triangle equals the product of $\sqrt{2}$ and the length of either leg, then the angles of the triangle measure 45° , 45° , and 90° .

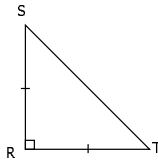
- How do we know the triangle is isosceles?
- Why does this mean the two acute angles are each 45° ?

Example

► $RS = RT$

► What are the measures of the angles of the triangle?

► If $RT = 12\sqrt{2}$, find RS .



30-60-90 Theorem

► **30-60-90 Theorem:** In a triangle whose angles measure 30° , 60° , and 90° , the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

► **Given**

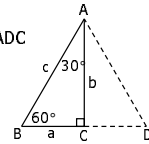
- Right $\triangle ABC$ with $m\angle A = 30^\circ$, $m\angle B = 60^\circ$, $BC = a$, $AC = b$, $AB = c$.

► **Prove**

- $c = 2a$ and $b = a\sqrt{3}$

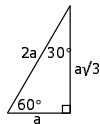
► **Construction**

- Extend BC to point D so that $BC = CD$; Make $\triangle ADC$
- Prove the two triangles are congruent
 - CPCTC shows the large triangle is equilateral
 - Equilateral \rightarrow Equilateral $\rightarrow c = 2a$
- Use Pythagorean Theorem
 - Get $b = a\sqrt{3}$



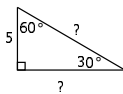
The 30-60-90 Triangle Examples

- Small side – opposite 30° angle – a
- Longest side – opposite 90° angle – $2a$
- Middle side – opposite 60° angle – $a\sqrt{3}$

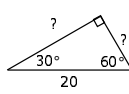


► Examples – Find the missing sides

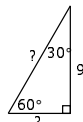
1.



2.



3.

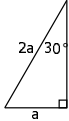


Example

► Each side of an equilateral triangle measures 6 in. Find the length of an altitude of the triangle.

Converse of 30-60-90 Theorem

- **Thm 5.4.4:** If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30° .



- **Ex:** In right $\triangle ABC$ with right $\angle C$, $AB = 24.6$ and $BC = 12.3$.
- What are the measures of the angles of the triangle?
 - Find AC .

Segments Divided Proportionally

- **Segments divided proportionally** means corresponding segments are proportional

► **Ex:**

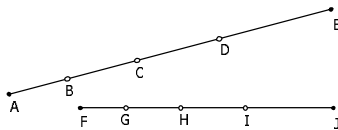
- D and E divide AB and AC proportionally.
- If $AD = 4$, $DB = 7$, and $EC = 6$, find AE .



New Way to Write Proportions

- If $a/b = c/d$, then $(a+c)/(b+d) = a/b = c/d$.

- **Ex:** Suppose AE and FJ are divided proportionally. If $AB/FG = 3$, find AC/FH .



- **Thm 5.5.1:** If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.

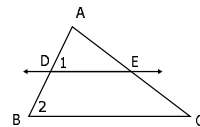
- Write proportions by subtraction:
- If $a/b = c/d$, then $(a-b)/b = (c-d)/d$.

- Given: $\triangle ABC$ with $AB \parallel DE$

- Prove: $AD/DC = BE/EC$

- Goals:

- Get triangles similar
- Use CSSTP on sides
- Write by subtraction

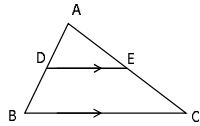


- **Ex:** $DE \parallel AB$, $AC = 24$, $CE = 3$, and $EB = 5$. Find the length of CD and DA .

Example

► Complete the proportions:

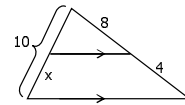
1. $AD/DB = \underline{\hspace{2cm}}$
2. $AD/AB = \underline{\hspace{2cm}}$
3. $CE/CA = \underline{\hspace{2cm}}$



Example

► Which of the following are correct proportions?

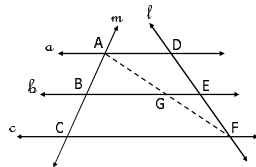
1. $(10 - x)/x = 8/4$
2. $x/10 = 4/8$
3. $x/(10 - x) = 4/8$
4. $x/10 = 4/12$



Cor 5.5.2: When three (or more) parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

► **Given:** $a \parallel b \parallel c$

► **Prove:** $AB/DE = BC/EF$



- Why does $AB/AG = BC/GF$?
 - Why can we write $AB/BC = AG/GF$?
- What proportion can you get from the $\triangle ADF$?
- How can you put the proportions together?

The Angle Bisector Theorem

► **5.5.3:** If a ray bisects one angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the two sides that form the bisected angle.

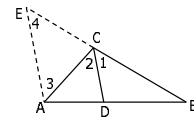
► **Given:** $\triangle ABC$, CD bisects $\angle ACB$

► **Prove:** $AD/DB = AC/CB$

- Segment at left/segment at right = side at left/side at right

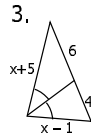
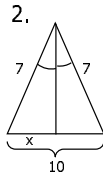
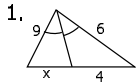
► **Steps:**

- Draw $EA \parallel DC$. Extend BC .
- Why is $EC/AD = CB/DB$?
- Why is $\angle 3 \cong \angle 4$?
- How does the conclusion follow?



Example

► Find x in each of the following:



Example

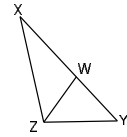
► YW bisects $\angle XYZ$

1. $XY = 3$, $YZ = 5$, and $XW = 2$.

▪ Find XZ .

2. $XY = 3$, $YZ = 4$, and $XZ = 5$.

▪ Find XW and WZ .



Homework

► Due Thursday 7/8

- Read Sections 5.4 and 5.5
- 5.4: #1-18, 23-28
- 5.5: #3-26