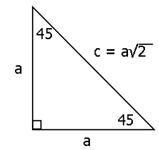
Math 119 – Plane Geometry

Sections 5.4 and 5.5
Similarity III
7/7/2004

45-45-90 Theorem

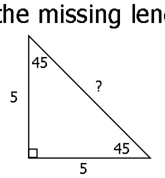
- **<u>45-45-90 Theorem</u>**: In a triangle whose angles measure 45°, 45°, and 90°, the hypotenuse has a length equal to the product of √2 and the length of either leg.

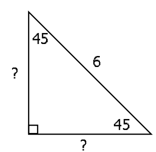
- <u>Given</u>: Isosceles right triangle with legs of measure a and hypotenuse of measure c
- <u>Prove</u>: $c = a\sqrt{2}$
- <u>Proof</u>: By Pythagorean Theorem



Examples

- Find the missing lengths:





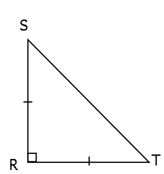
- A square has a diagonal of length $2\sqrt{2}$. Find the length of its sides.

Converse of 45-45-90 Theorem

- **<u>Thm 5.4.3</u>**: If the length of the hypotenuse of a right triangle equals the product of √2 and the length of either leg, then the angles of the triangle measure 45°, 45°, and 90°.
  - How do we know the triangle is isosceles?
  - Why does this mean the two acute angles are each 45°?

Example

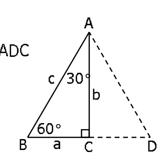
- RS = RT
- What are the measures of the angles of the triangle?
- If $RT = 12\sqrt{2}$, find RS.



30-60-90 Theorem

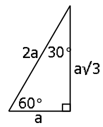
- **<u>30-60-90 Theorem</u>**: In a triangle whose angles measure 30°, 60°, and 90°, the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of √3 and the length of the shorter leg.

- <u>Given</u>
  - Right ΔABC with m∠A − 30°, m∠B − 60°. BC − a, AC − b, AB − c.
- <u>Prove</u>
  - $c - 2a$ and $b - a\sqrt{3}$
- <u>Construction</u>
  - Extend BC to point D so that BC − CD; Make ΔADC
  - Prove the two triangles are congruent
    - CPCTC shows the large triangle is equilangular
    - Equiangular → Equilateral → $c = 2a$
  - Use Pythagorean Theorem
    - Get $b = a\sqrt{3}$



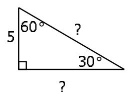
The 30-60-90 Triangle Examples

- Small side – opposite 30° angle – a
- Longest side – opposite 90° angle – 2a
- Middle side – opposite 60° angle - a√3



- Examples – Find the missing sides

1.



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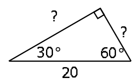

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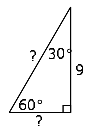
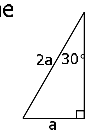


Example

- Each side of an equilateral triangle measures 6 in. Find the length of an altitude of the triangle.

Converse of 30-60-90 Theorem

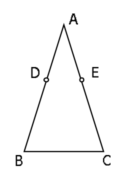
- **Thm 5.4.4**: If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30°.



- **Ex**: In right ΔABC with right ∠C, AB = 24.6 and BC = 12.3.
  - What are the measures of the angles of the triangle?
  - Find AC.

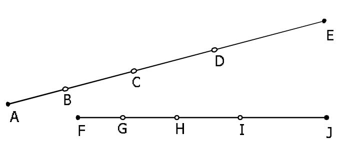
Segments Divided Proportionally

- **Segments divided proportionally** means corresponding segments are proportional
  - **Ex**:
    - D and E divide AB and AC proportionally.
    - If AD = 4, DB = 7, and EC = 6, find AE.



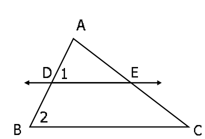
New Way to Write Proportions

- If a/b = c/d, then (a+c)/(b+d) = a/b = c/d.
- **Ex**: Suppose AE and FJ are divided proportionally. If AB/FG = 3, find AC/FH.



Thm 5.5.1: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.

- Write proportions by subtraction:
  - If a/b = c/d, then (a-b)/b = (c-d)/d.
- Given: ΔABC with AB ∥ DE
- Prove: AD/DC = BE/EC
- Goals:
  - Get triangles similar
  - Use CSSTP on sides
  - Write by subtraction

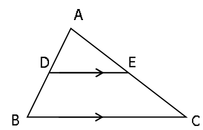


- **Ex**: DE ∥ AB, AC = 24, CE = 3, and EB = 5. Find the length of CD and DA.

Example

- Complete the proportions:

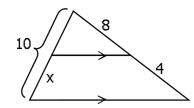
1. AD/DB = ____
2. AD/AB = ____
3. CE/CA = ____



Example

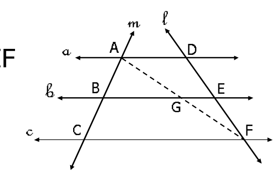
- Which of the following are correct proportions?

1. (10 – x)/x = 8/4
2. x/10 = 4/8
3. x/(10 – x) = 4/8
4. x/10 = 4/12



Cor 5.5.2: When three (or more) parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

- Given: $a \parallel b \parallel c$
- Prove: AB/DE = BC/EF

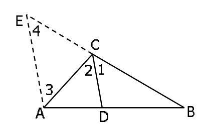


- Why does AB/AG = BC/GF?
  - Why can we write AB/BC = AG/GF?
- What proportion can you get from the ΔADF?
- How can you put the proportions together?

The Angle Bisector Theorem

- **5.5.3**: If a ray bisects one angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the two sides that form the bisected angle.

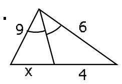
- Given: ΔABC, CD bisects ∠ACB
- Prove: AD/DB = AC/CB
  - Segment at left/segment at right = side at left/side at right
- Steps:
  - Draw EA ∥ DC. Extend BC.
  - Why is EC/AD = CB/DB?
  - Why is ∠3 ≅ ∠4?
  - How does the conclusion follow?



Example

- Find x in each of the following:

1.



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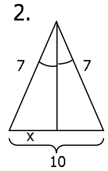

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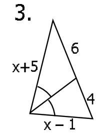
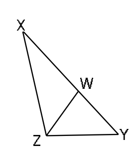


Example

- YW bisects ∠XYZ

1. XY = 3, YZ = 5, and XW = 2.
   - Find XZ.
2. XY = 3, YZ = 4, and XZ = 5.
   - Find XW and WZ.



Homework

- Due Thursday 7/8
  - Read Sections 5.4 and 5.5
  - 5.4: #1-18, 23-28
  - 5.5: #3-26