

Math 119 – Plane Geometry

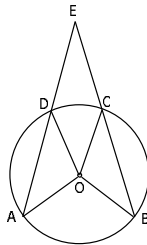
Sections 6.2 and 6.3
Circles II
7/12/2004

Recap For Measuring Angles

Location of Vertex	Measure of Angle
Center of circle	Measure of Intercepted Arc
Interior of Circle	$\frac{1}{2}$ Sum of Intercepted Arcs
Exterior of Circle	$\frac{1}{2}$ Difference of Intercepted Arcs
On Circle	$\frac{1}{2}$ Intercepted Arc

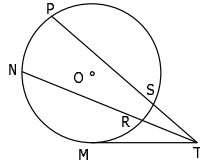
Example

- ▶ $\odot O$
- ▶ $m\angle AOB = 136^\circ$
- ▶ $m\angle DOC = 46^\circ$
- ▶ Find: $m\angle E$



Example

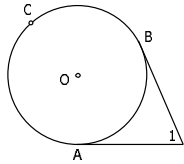
- ▶ $m\widehat{MN} = 70^\circ$
- ▶ $m\widehat{NP} = 88^\circ$
- ▶ $m\widehat{MR} = 46^\circ$
- ▶ $m\widehat{RS} = 26^\circ$



- ▶ Find:
 1. $m\angle MTN$
 2. $m\angle NTP$
 3. $m\angle MTP$

Example

- ▶ $m\angle 1 = 46^\circ$

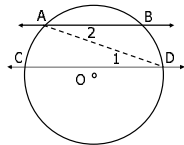


- ▶ Find $m\widehat{AB}$ and $m\widehat{ABC}$
 - Hint: Consider $m\widehat{AB} + m\widehat{ABC}$

- ▶ **Q:** What if \overline{AB} formed a diameter? What would be the measure of $\angle 1$?

Thm 6.2.8: If two parallel lines intersect a circle, the intercepted arcs between these lines are congruent. (HW)

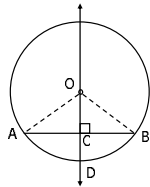
- ▶ **Given:** $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
- ▶ **Prove:** $\widehat{AC} \cong \widehat{BD}$



- ▶ **Helpful Hints:**
 - Why are angles 1 and 2 congruent?
 - Express each in terms of the intercepted arc.

Perpendicular Lines Through Center

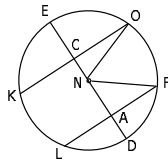
- ▶ **Recall:** A radius that is perpendicular to a chord bisects the chord.
- ▶ **Given:** $OD \perp AB$ in $\odot O$
- ▶ **Prove:** $\overline{CA} \cong \overline{CB}$ and $\widehat{CA} \cong \widehat{CB}$
- ▶ **Use:** HL and cpctc



- ▶ **Thm 6.3.1:** If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc. (HW)

Example

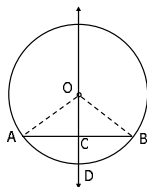
- ▶ In $\odot N$, $\overline{DE} \perp \overline{FL}$ and $\overline{DE} \perp \overline{OK}$; $DE = 50$, $FL = 30$, and $OK = 48$
- 1. Why does \overline{DE} bisect \overline{FL} and \overline{OK} ?
- 2. Find the following:
 - a. NF
 - b. NA
 - c. NC
- 3. If two chords have different lengths, which chord is closer to the center?



Line Through Center Bisect Chord

- ▶ **Thm 6.3.2:** If a line through the center of a circle bisects a chord other than a diameter, then it is perpendicular to the chord. (HW)

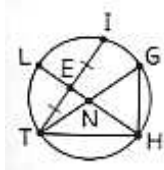
- ▶ **Given:** \overline{OD} bisects \overline{AB} in $\odot O$
- ▶ **Prove:** $\overline{AB} \perp \overline{OD}$
- ▶ **Use:** SSS and cpctc



- ▶ **Q:** Why can't the chord be a diameter?

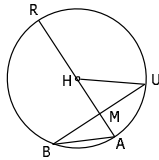
Example

- Why is $LH \perp TI$?
- Why does $m\widehat{GH} = m\angle GNH$?
- Why is $LN = NG$?
- Why is $\angle THG$ a right angle?



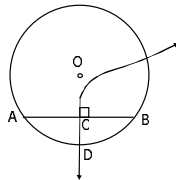
Example

- In $\odot H$, RA bisects BU ; $HU = 5$ and $HM = 3$
- Why is $RA \perp BU$?
 - Find the following
 - RA
 - BU
 - BA
 - MU
 - MA



Thm 6.3.3: The perpendicular bisector of a chord contains the center of the circle.

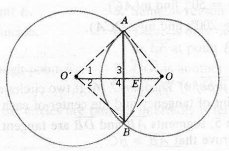
- Given: \overleftrightarrow{DC} is the perpendicular bisector of \overline{AB} in $\odot O$
- Prove: \overleftrightarrow{DC} contains point O



- Indirect Proof
- Assume: \overleftrightarrow{DC} does not contain O
 - Draw \overline{OC}
 - Why is $\overline{OC} \perp \overline{AB}$?
 - Why is this a contradiction?
 - What does this tell us about the assumption?

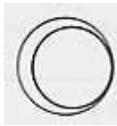
Example

- Circles O and O' intersect at A and B .
1. Is $\overline{OO'}$ the perpendicular bisector of \overline{AB} ?
 2. Is \overline{AB} the perpendicular bisector of $\overline{OO'}$?

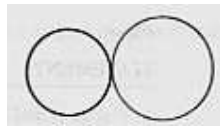


Circles That Are Tangent

- a. If two circles touch at one point, they are **tangent circles**.
- b. If one circle is on the interior of the other, except for a point of tangency, the circles are **tangent internally**.

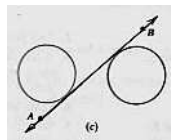


- c. Otherwise, they are **tangent externally**.

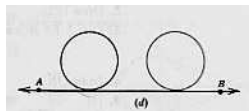


Common Tangents

- a. A tangent is a **common internal tangent** if it is tangent to 2 circles on opposite sides of it.



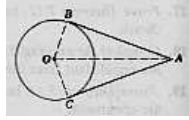
- b. A tangent is a **common external tangent** if it is tangent to 2 circles on the same side of it.



Thm 6.3.4: The tangent segments to a circle from an external point are congruent.

► **Given:** \overline{AB} and \overline{AC} are tangents to circle O

► **Prove:** $\overline{AB} = \overline{AC}$



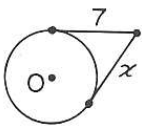
► **Steps:**

- Draw radii \overline{OB} and \overline{OC} . Also draw segment \overline{OA} .
- Use HL and cpctc

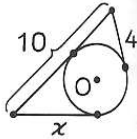
Example

► Tangent segments have been drawn to the circles in the figures. Find x in each of the following:

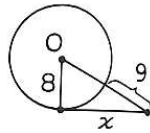
1.



2.



3.



Example

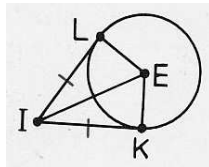
► \overline{IK} is tangent to circle E and $IL = IK$

a. Why is $\triangle ILE \cong \triangle IKE$?

b. Why is $\angle K$ a right angle?

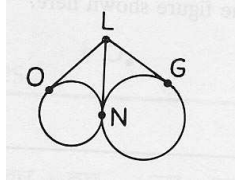
c. Why is $\angle L$ a right angle?

d. Why is \overline{IL} tangent to $\odot E$?



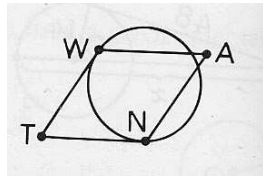
Example: A Proof

- ▶ **Given:** \overline{LO} , \overline{LN} , and \overline{LG} are tangent segments to the circles
- ▶ **Prove:** $\overline{LO} \cong \overline{LG}$



Example: Another Proof

- ▶ **Given:** \overline{TW} and \overline{TN} are tangent to the circle; $WANT$ is a parallelogram
- ▶ **Prove:** $WANT$ is a rhombus



The Intersecting Chords Theorem

▶ **Theorem 6.3.5:** If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.

▶ **Given:** Chords \overline{AB} and \overline{CD} intersect at point P in the circle

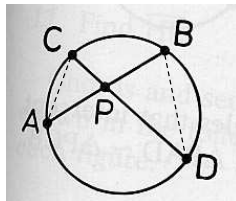
▶ **Prove:** $PA \cdot PB = PC \cdot PD$

▶ **Note:**

- $m\angle A \cong m\angle C$ and $m\angle D \cong m\angle B$
- Why?
- What arc does each intercept?

▶ **Q:** What does having two angles congruent get us?

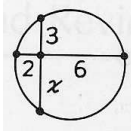
▶ Apply CSSTP.



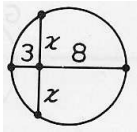
Example

► Chords have been drawn in the following figures. Find x .

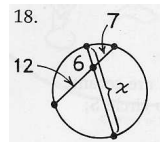
1.



2.



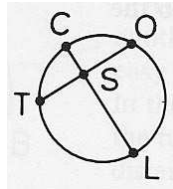
3.



Example

► Chords \overline{CL} and \overline{TO} intersect at point S in the circle

- Write an equation for this figure that follows from the Intersecting Chords Theorem.
- Use one of the theorems from last time to write a formula for $m\angle OSL$.
- If \overline{CL} is a diameter of the circle and bisects \overline{TO} , what can you conclude?

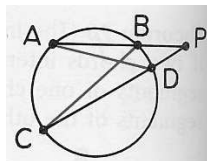


The Secant Segments Theorem

► **Theorem 6.3.6:** If two secants are drawn to a circle from an external point, then the products of the lengths of each secant with its external segment are equal.

- **Given:** The circle with secants \overline{PA} and \overline{PC}
- **Prove:** $PA \cdot PB = PC \cdot PD$

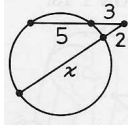
- **Draw:** \overline{CB} and \overline{AD}
- **Note:** $\angle A \cong \angle C$. Why?
- **Q:** Why can we now conclude $\triangle PAD \sim \triangle PCB$?
- Use CSSTP



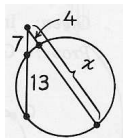
Example

► Secants have been drawn in the following figures. Find x .

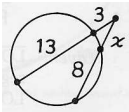
1.



2.

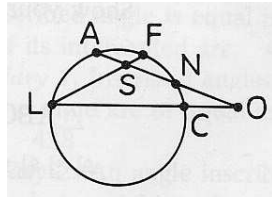


3.



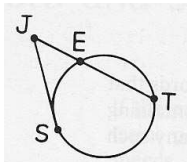
Example

- \overline{AO} and \overline{LO} are secant segments to the circle and \overline{LF} is a chord.
- $FS = 4$, $SL = 15$, $LC = 24$, $CO = 9$, and $NO = 11$.
- a. Find AO
- b. Find AN
- c. Find AS



Related Theorem in Your Book

- We won't prove this in class, but...
- **Theorem 6.3.7:** If a secant and a tangent are drawn to a circle, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.



Homework

► Due Tuesday 7/13

- Read Sections 6.2 and 6.3
- 6.2: #6, 7, 10, 11, 17-22, 31, 34, 35, 39
- 6.3: #1-5, 7-25, 38, 39, 40
