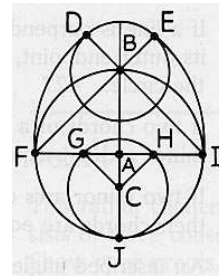


Math 119 – Plane Geometry

Sections 6.4 and 6.5
Circles III
7/13/2004

Review of Circle Vocabulary

- ▶ With Respect to Circle A:
 1. What is \overline{FI} called?
 2. What is \widehat{FBI} ?
 3. What is $\angle FBI$ called?
- ▶ With Respect to circle B:
 4. What is \overline{BE} called?
 5. What is \widehat{DE} called?
 6. What is $\angle DBE$ called?
 7. What is \overline{FE} called?
- ▶ With respect to circle C:
 8. What is \overline{GH} called?
 9. What is \widehat{GJH} called?
 10. What is $\angle HAJ$ called?

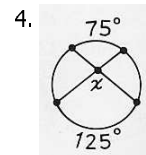
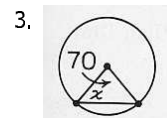
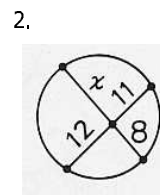
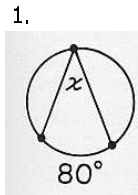


True or False:

1. A diameter is a chord.
2. The measure of a minor arc is less than 90° .
3. If a line through the center of a circle bisects a chord, it must also be perpendicular to it.
4. An inscribed angle can intercept a major arc.
5. A secant is a line that intersects a circle in more than one point.
6. If a line is perpendicular to a radius of a circle, it must be tangent to the circle.
7. Two central angles in a circle are equal if and only if their minor arcs are equal.

Examples - Using Circle Theorems

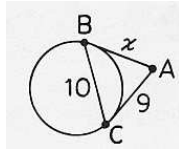
▶ Solve for x.



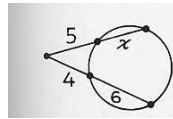
More Examples – Using Circle Theorems

► Solve for x .

1. \overline{AB} and \overline{AC} are tangents



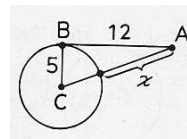
2.



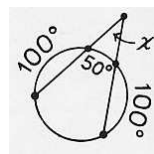
Even More Circle Examples

► Solve for x .

1. \overline{AB} is tangent to circle C



2.



Tangents Perpendicular to Radii

► **Thm 6.4.1:** The line that is perpendicular to the radius of a circle at its endpoint on the circle is a tangent to the circle.

- Indirect Proof
- Assume the line is not tangent
- Draw the tangent
 - Must be perpendicular
- Why is 2 separate lines perpendicular to the same line through the same point a contradiction?

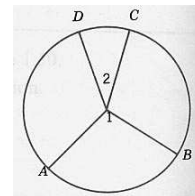
► **Construction:** Tangent to circle at a point on the circle.

- Make perpendicular line to the radius at the point

Inequalities in Circles

► **Theorem 6.4.2:** In a circle, the larger of two central angles will intercept the larger arc.

- **Given:** Angles 1 and 2 are central angles with $m\angle 1 > m\angle 2$
- **Prove:** $\widehat{AB} > \widehat{CD}$
- **Q:** How do we measure arcs?

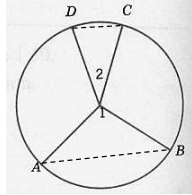


► **Theorem 6.4.3:** In a circle containing two unequal arcs, the larger arc corresponds to the larger central angle. (HW)

- **Given:** $\widehat{AB} > \widehat{AC}$
- **Prove:** $m\angle 1 > m\angle 2$

Example

a. If $\widehat{AB} = 120^\circ$ and $\widehat{DC} = 40^\circ$, which of $m\angle 1$ and $m\angle 2$ is larger?



b. If $m\angle 1 = 45^\circ$ and $m\angle 2 = 50^\circ$, which of \widehat{AB} and \widehat{CD} is the larger?

c. What does intuition suggest about DC and AB?

Application of The Hinge Theorem to Circles

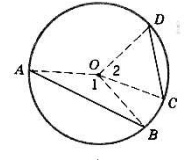
► **Theorem 6.4.6:** In the same circle, the longer of two chords intercepts the greater **minor** arc. (HW)

▪ **Given:** $AB > CD$

▪ **Prove:** $m\widehat{AB} > m\widehat{CD}$

▪ **Proof:**

- Why are \overline{OA} , \overline{OB} , \overline{OC} , and \overline{OD} all congruent?
- How can we apply the Hinge Theorem to get $m\angle 1 > m\angle 2$?
- Why does that make $m\widehat{AB} > m\widehat{CD}$?



► **Theorem 6.4.7:** In the same circle, the greater of two **minor** arcs has the longer chord. (In book)

Example – True/False (Be Careful!)

- The greater of two arcs in a circle has the greater chord.
- If a central angle measures 60° , then its intercepted chord is longer than the radius of the circle.

Chords Inequalities

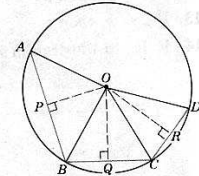
► **Theorem 6.4.4:** In a circle containing two unequal chords, the shorter chord is at the greater distance from the center of the circle.

► **Theorem 6.4.5:** In a circle containing two unequal chords, the chord nearer the center of the circle has the greater length.

► **Example:**

$m\widehat{AB} > m\widehat{BC} > m\widehat{CD}$

- Write AB, BC, and CD in increasing order.
- Write OP, OQ, and OR in decreasing order.



Major Question in Mathematics: How to Define an Object

- ▶ Is this the only way to define this?
- ▶ How else might I define this?
- ▶ Is this definition enough to define the object?
- ▶ Is everything in this definition necessary for the definition?
- ▶ Does this definition help me intuitively see the object I am defining?

Geometric Loci

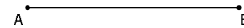
- ▶ A **locus** is the set of all points and only those points that satisfy a given condition (or set of conditions).
 1. All points in the locus satisfy a given condition, and
 2. all points satisfying the given condition are in the locus.
- ▶ Locus is derived from the same root words as location
- ▶ Understanding loci:
 - Start with a set of conditions; determine what shape the object is

New Look at the Circle

- ▶ What is it about a circle that makes it a circle?
- ▶ Think of the shape of something by thinking about the path a ladybug takes when walking along this thing.
- ▶ What locus makes a circle?

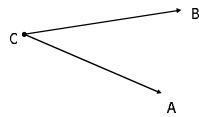
Locus of Points Equidistant from the Ends of a Segment

- ▶ What can you say about the midpoint of segment AB?
 - Is it a point on the locus?
 - What are the other points?



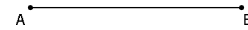
Locus of Points Equidistant from the Sides of an Angle

- ▶ In an acute angle, find a few points that are equidistant from both rays of the angle.
- ▶ What do you think the locus of points equidistant from these two rays defines?



Locus of the Vertex of the Right Angle of a Right Triangle with Fixed Hypotenuse

- ▶ Draw a right triangle with hypotenuse \overline{AB} .
- ▶ What parts of the description are not moving/changing?
- ▶ What parts are moving/changing?
- ▶ Find a few points in the locus.
- ▶ In a circle, what kind of inscribed angle always has a semicircle for an intercepted arc?
 - What is the measure of this inscribed angle?



Example: Find the Locus

- What is the locus of all points in a plane that are 3 units from a given point in that plane?
- What is the locus of all points in a plane that are 7 units from a line in that plane?
- What is the locus of all points in a plane that are equidistant from two perpendicular lines in that plane?

Homework

- ▶ Due Wednesday 7/14
 - Read Sections 6.4 and 6.5
 - 6.4: #1-8, 13-25, 30, 31, 32
 - 6.5: #1-17, 32