Math 119 – Plane Geometry

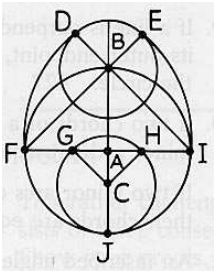
Sections 6.4 and 6.5

Circles III

7/13/2004

Review of Circle Vocabulary

- With Respect to Circle A:
  1. What is $\overline{FI}$ called?
  2. What is $\overset{\frown}{FBI}$?
  3. What is $\angle FBJ$ called?
- With Respect to circle B:
  4. What is $\overline{BE}$ called?
  5. What is $\overset{\frown}{DE}$ called?
  6. What is $\angle DBE$ called?
  7. What is $\overline{FE}$ called?
- With respect to circle C:
  8. What is $\overline{GH}$ called?
  9. What is $\overset{\frown}{GJH}$ called?
  10. What is $\angle HAJ$ called?



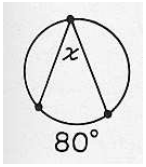
True or False:

1. A diameter is a chord.
2. The measure of a minor arc is less than 90.
3. If a line through the center of a circle bisects a chord, it must also be perpendicular to it.
4. An inscribed angle can intercept a major arc.
5. A secant is a line that intersects a circle in more than one point.
6. If a line is perpendicular to a radius of a circle, it must be tangent to the circle.
7. Two central angles in a circle are equal if and only if their minor arcs are equal.

Examples - Using Circle Theorems

► Solve for x.

1.



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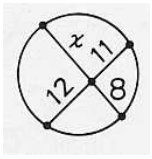

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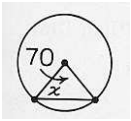

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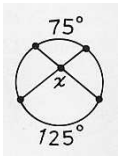
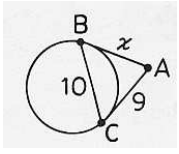


More Examples – Using Circle Theorems

► Solve for x.

1. $\overline{AB}$ and $\overline{AC}$ are tangents



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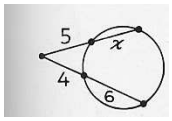
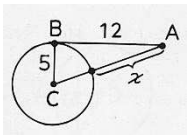


Even More Circle Examples

► Solve for x.

1. $\overline{AB}$ is tangent to circle C



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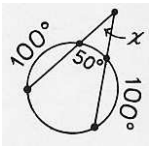
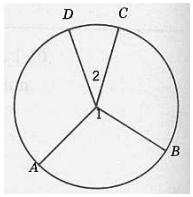


Tangents Perpendicular to Radii

- **Thm 6.4.1**: The line that is perpendicular to the radius of a circle at its endpoint on the circle is a tangent to the circle.
  - Indirect Proof
  - Assume the line is not tangent
  - Draw the tangent
    - Must be perpendicular
  - Why is 2 separate lines perpendicular to the same line through the same point a contradiction?
- **Construction**: Tangent to circle at a point on the circle.
  - Make perpendicular line to the radius at the point

Inequalities in Circles

- **Theorem 6.4.2**: In a circle, the larger of two central angles will intercept the larger arc.
  - **Given**: Angles 1 and 2 are central angles with $m\angle 1 > m\angle 2$
  - **Prove**: $\overset{\frown}{AB} > \overset{\frown}{CD}$
  - **Q**: How do we measure arcs?

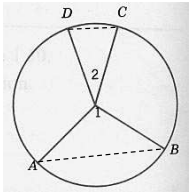


- **Theorem 6.4.3**: In a circle containing two unequal arcs, the larger arc corresponds to the larger central angle. (HW)
  - **Given**: $\overset{\frown}{AB} > \overset{\frown}{AC}$
  - **Prove**: $m\angle 1 > m\angle 2$

Example

a. If $\overset{\frown}{AB} = 120°$ and $\overset{\frown}{DC} = 40°$, which of $m\angle 1$ and $m\angle 2$ is larger?

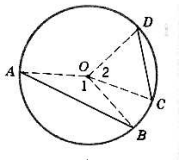
b. If $m\angle 1 = 45°$ and $m\angle 2 = 50°$, which of $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ is the larger?



c. What does intuition suggest about DC and AB?

Application of The Hinge Theorem to Circles

- **Theorem 6.4.6**: In the same circle, the longer of two chords intercepts the greater **minor** arc. (HW)
  - **Given**: AB > CD
  - **Prove**: $m\overset{\frown}{AB} > m\overset{\frown}{CD}$
  - **Proof**:
    a. Why are $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$ all congruent?
    b. How can we apply the Hinge Theorem to get $m\angle 1 > m\angle 2$?
    c. Why does that make $m\overset{\frown}{AB} > m\overset{\frown}{CD}$?



- **Theorem 6.4.7**: In the same circle, the greater of two **minor** arcs has the longer chord. (In book)

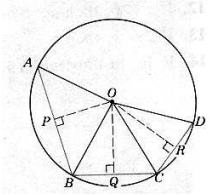
Example – True/False (Be Careful!)

a. The greater of two arcs in a circle has the greater chord.

b. If a central angle measures 60, then its intercepted chord is longer than the radius of the circle.

Chords Inequalities

- **Theorem 6.4.4**: In a circle containing two unequal chords, the shorter chord is at the greater distance from the center of the circle.
- **Theorem 6.4.5**: In a circle containing two unequal chords, the chord nearer the center of the circle has the greater length.
- **Example**: $m\overset{\frown}{AB} > m\overset{\frown}{BC} > m\overset{\frown}{CD}$
  a. Write AB, BC, and CD in increasing order.
  b. Write OP, OQ, and OR in decreasing order.



Major Question in Mathematics: How to Define an Object

- Is this the only way to define this?
- How else might I define this?
- Is this definition enough to define the object?
- Is everything in this definition necessary for the definition?
- Does this definition help me intuitively see the object I am defining?

Geometric Loci

- A **locus** is the set of all points and only those points that satisfy a given condition (or set of conditions).
  1. All points in the locus satisfy a given condition, and
  2. all points satisfying the given condition are in the locus.
- Locus is derived from the same root words as location
- Understanding loci:
  - Start with a set of conditions; determine what shape the object is

New Look at the Circle

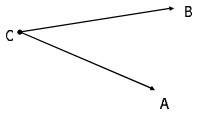
- What is it about a circle that makes it a circle?
- Think of the shape of something by thinking about the path a ladybug takes when walking along this thing.
- What locus makes a circle?

Locus of Points Equidistant from the Ends of a Segment

- What can you say about the midpoint of segment AB?
  - Is it a point on the locus?
  - What are the other points?

Locus of Points Equidistant from the Sides of an Angle

- In an acute angle, find a few points that are equidistant from both rays of the angle.
- What do you think the locus of points equidistant from these two rays defines?



Locus of the Vertex of the Right Angle of a Right Triangle with Fixed Hypotenuse

- Draw a right triangle with hypotenuse $\overline{AB}$.
- What parts of the description are not moving/changing?
- What parts are moving/changing?
- Find a few points in the locus.
- In a circle, what kind of inscribed angle always has a semicircle for an intercepted arc?
  - What is the measure of this inscribed angle?

Example: Find the Locus

a. What is the locus of all points in a plane that are 3 units from a given point in that plane?
b. What is the locus of all points in a plane that are 7 units from a line in that plane?
c. What is the locus of all points in a plane that are equidistant from two perpendicular lines in that plane?

Homework

- Due Wednesday 7/14
  - Read Sections 6.4 and 6.5
  - 6.4: #1-8, 13-25, 30, 31, 32
  - 6.5: #1-17, 32