Math 119 – Plane Geometry

Sections 6.6 and 7.1

Circles IV and Area I

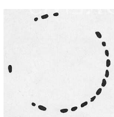
7/14/2004

Concurrency Theorems

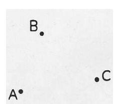
- A number of lines are **<u>concurrent</u>** if they have exactly one point in common.
- Concurrent parts of a triangle:
  - Angle Bisectors
  - Perpendicular Bisectors of the Sides
  - Altitudes
  - Medians

Stonehenge

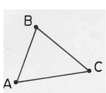
- First: Picture of stones that still stand in the main circle



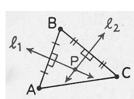
- The center can be determined from the positions of any three of its stones.



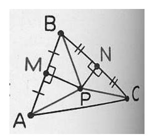
1. First form a triangle.



2. Let ℓ_1 and ℓ_2 be perpendicular bisectors

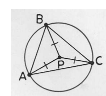


3. This leaves us with:



- Why does PA = PB and PB = PC?

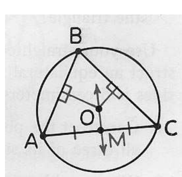
4. What figure have we made?
   - **<u>Circumcircle</u>**
   - The center is called the **<u>circumcenter</u>** of the triangle.



- A polygon is **<u>cyclic</u>** if, and only if, there exists a circle that contains all of its vertices.
- We just showed: **Every triangle is cyclic**.
- A bit ago we showed the conditions for a quadrilateral to be cyclic. What were they?

Corollary to What We Just Showed

- **<u>Thm 6.6.2</u>**: The three perpendicular bisectors of the sides of a triangle are concurrent.
  - We've already shown this for two of the perpendicular bisectors
  - Consider the midpoint of the third side of the triangle.
    - Why does this make OM a *perpendicular* bisector?
    - (Hint: SSS)



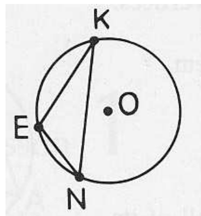
Construction of the Circle

- Construct the perpendicular bisectors of 2 sides
- Use this to make the circle
- Why do we know this is a circle with the vertices of the triangle on it?

Example

- The vertices of ΔKEN are on circle O

a. What are $\overline{KE}$, $\overline{EN}$ and $\overline{KN}$ called with respect to the circle?
b. What are angles K, E, and N called with respect to the circle?
c. What relation do points K, E, and N have to point O?
d. What is circle O called with respect to ΔKEN?
e. What is point O called with respect to the triangle?



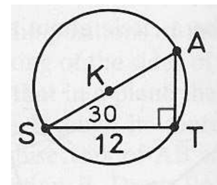
Example

- SAT is a 30-60-90 triangle inscribed in ⊙K
- ST = 12
- Find:

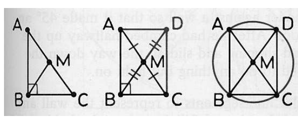
a. m$\widehat{ST}$

b. AT

c. KA



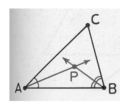
Example: Circumcenter of a Right Triangle



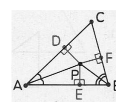
- In the first figure, M is the midpoint of the hypotenuse of right triangle ABC
  - Why does MA = MC?
- In the second figure, point D has been chosen on $\overline{BM}$ so that MD − BM. $\overline{DA}$ and $\overline{DC}$ have been drawn.
  - Why is ABCD a parallelogram?
  - What special type of parallelogram is ABCD?
  - Why does BD = AC
- Because MB − ½ BD and MA − MC − ½ AC, it follows that MB − MA − MC.
  - Why can a circle be drawn with center M and radius MA that goes through points A, B, and C?

Thm 6.6.1: The three angle bisectors of a triangle are concurrent.

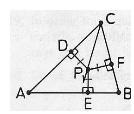
1. Let ABC be any triangle. Let two rays bisect angles A and B, respectively, and intersect at P



2. Draw perpendiculars from P to the sides of the triangle.
   - ΔPAD ≅ ΔPAE, ΔPBE ≅ ΔPBF by AAS
   - Why does PD = PE = PF?



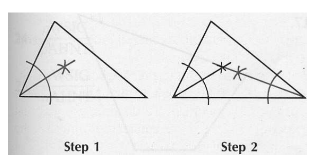
3. Draw CP.
   - ΔPCD ≅ ΔPCF by HL
   - Why does $\overrightarrow{CP}$ bisect C?

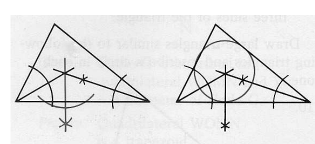


Vocabulary

- The point of intersection of the angle bisectors is called the **incenter of the triangle**.
- The circle that is tangent to all three sides of the triangle is called the **incircle**.
- Why is the incenter the center of an inscribed circle?

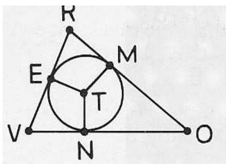
Construction of the Circle





Example

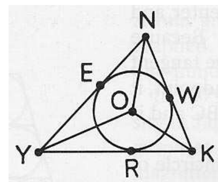
- Circle T is inscribed in ΔVRO



a. What relation do the sides of the triangle have to the circle?
b. What relation do the radii have to the sides? Why?
c. Why is TE = TM = TN?
d. What is circle T called with respect to the triangle?
e. What is point T called with respect to the triangle?

Example

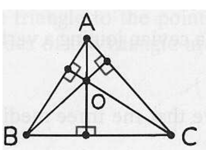
- Circle O is inscribed in ΔNYK

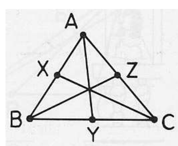


a. Why is NE = NW, YE = YR, and KW = KR?
b. What relation do $\overline{NO}$, $\overline{YO}$, and $\overline{KO}$ have to the angles of ΔNYK?
c. What relation does point O have to the three sides of the triangle?

More Concurrency Theorems

- **Thm 6.6.3**: The three altitudes of a triangle (or their extensions) are concurrent.
- Point of Intersection – **orthocenter**

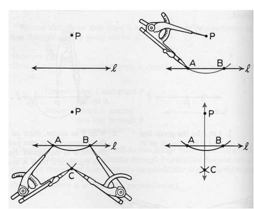




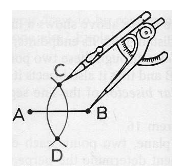
- **Thm 6.6.4**: The three medians of a triangle are concurrent at a point that is 2/3 the distance from any vertex to the midpoint of the opposite side.
- Point of intersection – **centroid**

Recall Constructions

- Altitude of a Triangle

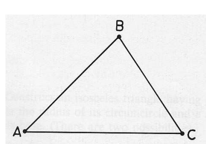


- Median of a Triangle



Review of Constructions

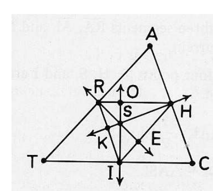
1. Circumscribe a circle about ΔABC
2. Inscribe a circle in it
3. Construct the point of concurrency of the medians
4. Construct the point of concurrency of the altitudes



Example

- $\overleftrightarrow{RE}$, $\overleftrightarrow{HK}$, and $\overleftrightarrow{IO}$ are the perpendicular bisectors of the sides of ΔATC

a. What is point S called with respect to ΔATC?
b. What are $\overline{RH}$, $\overline{HI}$, and $\overline{IR}$ called with respect to ΔATC?
c. Why is each side of ΔRHI parallel to one of the sides of ΔATC?
d. Why is $\overleftrightarrow{RE}$ perpendicular to $\overline{IH}$ (and $\overleftrightarrow{HK}$ to $\overline{RI}$; $\overleftrightarrow{IO}$ to $\overline{RH}$)?
e. What are $\overleftrightarrow{RE}$, $\overleftrightarrow{HK}$, and $\overleftrightarrow{IO}$ called with respect to ΔRHI?
f. What is point S called with respect to ΔRHI?

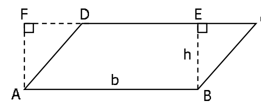


Initial Area Postulates

- **Area Postulate**: Corresponding to every bounded region is a unique positive number A, known as the area of that region.
  - Measured in square units
- **Post 19**: If two closed plane figures are congruent, then their areas are equal.
- **Area-Addition Postulate**: Let R and S be two enclosed regions that do not overlap. Then $A_{R \cup S} = A_R + A_S$.
- **Post 21**: The area A of a rectangle whose bas has length b and whose altitude has length h is given by A = bh.
- **Thm 7.1.1**: The area A of a square whose sides are each of length s is given by $A = s^2$.

Area of a Parallelogram

- **Recall**: An **altitude** of a parallelogram is a perpendicular segment from one side to the opposite side, known as the **base**.
- **Thm 7.1.2**: The area A of a parallelogram with a base of length b and with corresponding altitude of length h is given by A = bh.
  - Construct $\overline{BE} \perp \overline{CD}$, $\overline{FA} \perp \overline{CD}$
  - ΔFAD ≅ ΔEBC
  - FEBA is a rectangle
  - Area of Rectangle
  - Area-Addition Postulate

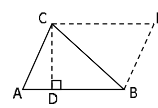


Example

1. If two parallelograms have equal perimeters, does it follow that they have equal areas?
2. If two parallelograms have equal areas, does it follow that they have equal perimeters?

Area of a Triangle

- **Thm 7.1.3**: The area A of a triangle whose base has length b and whose corresponding altitude has length h is given by A = ½ bh.
  - Construct EC ∥ AB, EB ∥ AC
  - Area of Parallelogram
  - Congruent Triangles -> Equal Area

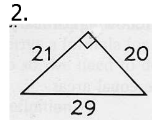


Example

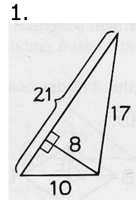
- Does perimeter relate to area? If two triangles have the same area, do they have the same perimeter? What if two pairs of sides are congruent?
- If the sides of a triangle are doubled, is the perimeter doubled?
- If the sides of a triangle are doubled, is the area doubled?

Example – Find the Area

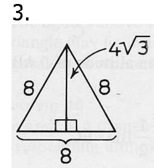
2.



1.

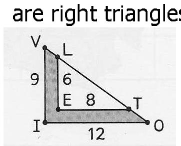


3.

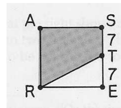


Example – Find Shaded Area

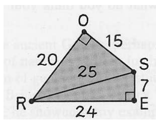
1. ΔVIO and Δ LET are right triangles



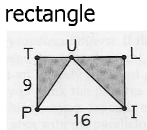
2. ASER is a square



3. $\overline{RO} \perp \overline{OS}$ and $\overline{RE} \perp \overline{ES}$

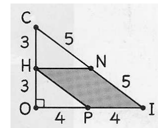


4. TLIP is a rectangle

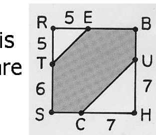


Example – Find Shaded Area

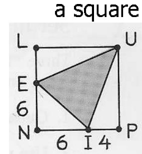
1. ΔCOI is a right triangle



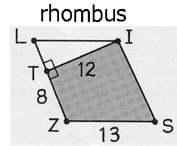
2. SHBR is a square



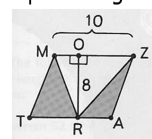
3. LUPN is a square



4. LISZ is a rhombus

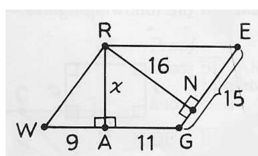


5. MZAT is a parallelogram



Example

- WGER is a parallelogram with altitudes $\overline{RA}$ and $\overline{RN}$
  1. Find A_{WGER}.
  2. Write an expression for the area in terms of x.
  3. Find x.
  4. Find the A_{AGER}.



Homework

- Due Thursday 7/15
  - Read Sections 6.6 and 7.1
  - 6.6: #1-17, 21-27
  - 7.1: #1-24, 30, 31, 34