

Math 119 – Plane Geometry

Sections 7.4 and 7.5
Area III
7/20/2004

Circumference

► **Post 22:** The ratio of the circumference of a circle to the length of its diameter is a unique positive constant.

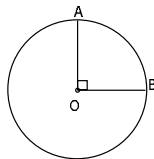
- Define π to be this ratio: $\pi = C/d$ in any circle
 - π is irrational
- Approximations: $22/7$, 3.14

► **Thm 7.4.1:** The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

Example

► In $\odot O$, $OA = 7$. Using $\pi \approx 22/7$

- Find the approximate circumference C of $\odot O$
- Find the approximate length of the minor arc AB



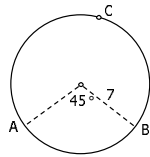
► The exact circumference of a circle is 17π .

- Find the length of the radius
- Find the length of the diameter

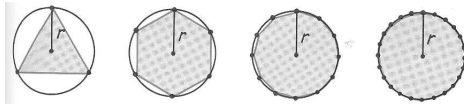
Length of An Arc

► **Thm 7.4.2:** In a circle whose circumference is C , the length l of an arc whose degree measure is m is given by $l = m/360 * C$.

► **Ex:** Find the approximate length of major arc ABC in a circle of radius 7 if $m\widehat{AC} = 45^\circ$. Use $\pi = 22/7$.



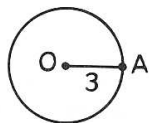
Area of a Circle



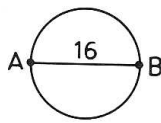
- As the number of sides of a regular polygon inscribed in a circle increases, the area of the polygon becomes a better and better approximation for the area of a circle.
- Recall: The **circumference of a circle** is the distance around the circle. $C = 2\pi r$
 - $A(\text{regular polygon}) = \frac{1}{2} Pr$
 - $A(\text{circle}) = \frac{1}{2} (C r) = \pi r^2$

Example

- a. A circle has an area of 64π . Find its diameter.
- b. Find the area of each of the following:



\overline{OA} is a radius.



\overline{AB} is a diameter.

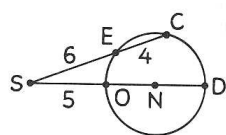
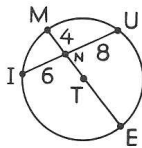
Example

- ▶ As the number of sides of a regular polygon inscribed in a circle increases,
 - a. What measurement of the circle do the perimeters of the polygons approach?
 - b. What measurement of the circle do the areas of the polygons approach?

Example

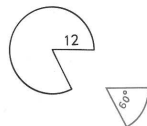
- ▶ \overline{IU} and \overline{ME} are chords of circle T.
- ▶ \overline{SC} and \overline{SD} are secants to circle N.

 - a. Find ME.
 - b. Find the area of circle T.
 - c. Find ND.
 - d. Find the area of circle N.



Sectors

- ▶ A **sector of a circle** is a region bounded by two radii of the circle and an arc intercepted by those radii.
 - (A slice of pizza)

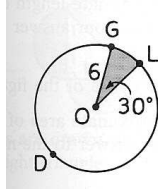


- ▶ **Post 23:** The ratio of the degree measure m of the central angle of a sector to 360° is the same as the ratio of the area of the sector to the area of the circle.
 - $A(\text{sector})/A(\text{circle}) = (\text{central angle of sector})/360$

Example

► In circle O, $OG = 6$ and $m\angle O = 30^\circ$.

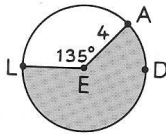
- Find the circumference of the circle.
- Find $m\widehat{GL}$.
- Find $m\widehat{GD}$.
- Find the area of the circle.
- Find the area of the shaded sector.



Example

► In circle E, $EA = 4$ and $m\angle E = 135^\circ$.

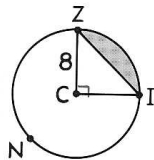
- Find the circumference of the circle.
- Find the $m\widehat{LDA}$.
- Find the area of the circle.
- Find the area of the shaded sector.



Segment Example

► A **segment of a circle** is the area enclosed by an arc and its respective chord.

- Find the area of the circle.
- Find the area of sector ZCI.
- Find the area of $\triangle ZCI$.
- Find the area of the shaded region (the segment).
- Find the area of the region bounded by \overline{ZI} and \widehat{ZNI} .

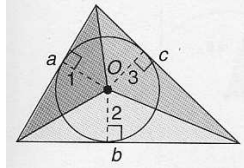


Area of A Triangle With an Inscribed Circle

► **Thm 7.5.3:** Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by

$$A = \frac{1}{2} rP.$$

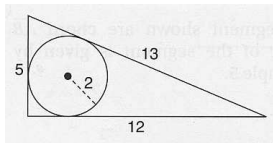
- Calculate area of each small triangle



Example

► Find the area of a triangle whose sides measure 5, 12 and 13 if the radius of the inscribed circle is 2.

- Use the latest theorem
- Use Heron's Formula
- Use formula for area of triangle



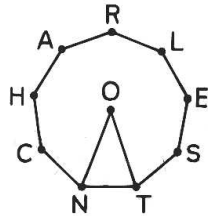
Review Example: True or False

- An apothem of a regular polygon bisects one of its sides.
- A circle can be circumscribed about a polygon if, and only if, the polygon is regular.
- Each central angle of a regular polygon having n sides has a measure of $360/n$.
- The ratio of the circumference to the diameter of a circle does not depend on the circle's size.
- The area of a circle is $1/4\pi d^2$, where d is the length of its diameter.

Example

► CHARLESTN is a regular nonagon with center O. Find the measure of each of the following angles:

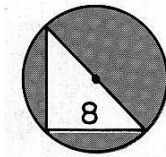
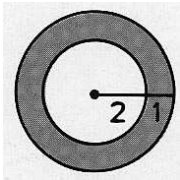
- a. $\angle O$
- b. $\angle ONT$
- c. $\angle CNT$



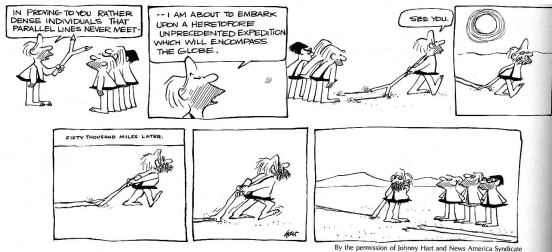
Example

► Find the area of the shaded region in each figure.

- a. The circles are concentric.
- b. An isosceles right triangle is inscribed in the circle.

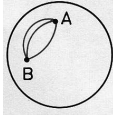


Things to Think About...



► Euclid defined parallel lines as "lines that, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction."

Great Circles



► Distance between two points

- On plane: Measured along the line determined by the two points
- On surface of sphere: **Q:** Which curved path to measure by? **A:** The shortest

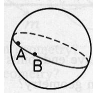
► A **great circle** is the locus of the intersection of the sphere and a plane containing the sphere's center.

► Lines on the Sphere

► Do 2 points determine a line on a sphere?

► Through P, how many lines can be drawn parallel to ℓ ?

- Parallel lines lie in same plane but do not intersect
- Consider sphere to be plane
- Parallel Postulate doesn't apply!



Homework

► Due Wednesday 7/21

- Read Sections 7.4 and 7.5
- 7.4: #1-12, 16, 21-31, 40
- 7.5: #1-18, 21, 25
